4.2 – #18:

$$\vec{F} = \frac{x^2}{y}\vec{i} + y\vec{j} + \vec{k}$$

(a) find flow lines: $\vec{R}(t) = \langle x, y, z \rangle$

$$\Rightarrow \frac{d\vec{R}}{dt} = \beta\vec{F} \Rightarrow \frac{dx}{dt} = \beta\frac{x^2}{y} \qquad \frac{dy}{dt} = \beta y \qquad \frac{dz}{dt} = \beta$$

$$\Rightarrow \quad \beta = \boxed{\frac{dx}{x^2/y} = dy/y = dz}$$

$$\Rightarrow \quad \frac{dx}{x^2/y} = dy/y \Rightarrow \int\frac{dx}{x^2} = \int\frac{dy}{y^2} \Rightarrow -\frac{1}{x} = -\frac{1}{y} + C_1 \qquad (1)$$

separate variables

also $$\frac{dy}{y} = dz \Rightarrow \int\frac{dy}{y} = \int dz \Rightarrow \ln y = z + C_2 \qquad (2)$$

general flow lines

through $(1,1,0)$: by (1) $\Rightarrow -1 = -1 + C_1 \Rightarrow C_1 = 0$

by (2) $\Rightarrow \ln(1) = 0 + C_2$, since $\ln(1) = 0$

$\Rightarrow C_2 = 0$ also.

so: $-\frac{1}{x} = -\frac{1}{y}$ OR $\boxed{x = y}$

and $\ln y = z$ OR $\boxed{y = e^z}$

flow line through $(1,1,0)$.

(b) does this pass through $(e,e,1)$? if so $\Rightarrow z = 1$

$\Rightarrow y = e^1 = e$ and $x = y = e$ ✓

(c)

$$\int_C \vec{F}\cdot d\vec{R} = ?$$ where C is the path along the flow line $x=y,\ y=e^z$ from $(1,1,0)$ to $(e,e,1)$.

parametrize the flow line:

let $z=t$: then $y=e^z \Rightarrow y=e^t$

and $x=y \Rightarrow x=e^t$

$$\Rightarrow \vec{R}(t) = \langle e^t, e^t, t\rangle.$$

so $$\vec{F}(\vec{R}(t)) = \left\langle \frac{(e^t)^2}{e^t},\ e^t,\ 1\right\rangle$$

and $$\frac{d\vec{R}}{dt} = \langle e^t, e^t, 1\rangle$$

$$\Rightarrow \int_{(1,1,0)}^{(e,e,1)} \vec{F}\cdot d\vec{R} = \int_0^1 \langle e^t, e^t, 1\rangle\cdot\langle e^t, e^t, 1\rangle\, dt$$

$$= \int_0^1 2e^{2t}+1\, dt = e^{2t}+t\Big|_0^1 = (e^2+1)-(e^0+0)$$

$$= e^2$$