3.6 – #4

Laplace's equation is $\Delta f = 0$.

Does f satisfy Laplace's eq'n?

(a)

$$f = e^z \sin y \Rightarrow \Delta f = \vec{\nabla}\cdot(\vec{\nabla} f) = \vec{\nabla}\cdot\left(\langle 0, e^z\cos y, e^z \sin y\rangle\right)$$

$$= -e^z \sin y + e^z \sin y = 0$$

$\Rightarrow \Delta f = 0$, so yes it does satisfy Laplace's eq'n

(b)

$$f = \sin x \sinh(y) + \cos x \cosh(z) = \sin x\left(\frac{e^y - e^{-y}}{2}\right) + \cos x\left(\frac{e^z + e^{-z}}{2}\right)$$

$$\Rightarrow \Delta f = \vec{\nabla}\cdot(\vec{\nabla} f) = \vec{\nabla}\cdot\left\langle \cos x \sinh y + -\sin x \cosh z,\ \sin x\left(\frac{e^y + e^{-y}}{2}\right),\ \cos x\left(\frac{e^z - e^{-z}}{2}\right)\right\rangle$$

$$= \vec{\nabla}\cdot\langle \cos x \sinh y - \sin x \cosh z,\ \sin x \cosh y,\ \cos x \sinh z\rangle$$

$$= (-\sin x \sinh y - \cos x \cosh z) + \sin x \sinh y + \cos x \cosh z$$

$$= 0 \checkmark$$

$\Rightarrow \Delta f = 0$ and f does satisfy Laplace's equation.

(c)

$$f(x, y, z) = \sin(px)\sinh(qy)$$

$$\Rightarrow \vec{\nabla}\cdot(\vec{\nabla} f) = \vec{\nabla}\cdot\left(\langle p\cos(px)\sinh(qy),\ \sin(px)(q\cosh(qy))\rangle\right)$$

$$= -p^2 \sin(px)\sinh(qy) + q^2 \sin(px)\sinh(qy)$$

$$= (q^2 - p^2)\sin(px)\sinh(qy)$$

thus $\Delta f = 0$ only if $q = p$.