MATH 513 – Vector Analysis

Name: _____

SSN: _____

(1) Let L_1 and L_2 be two lines given by

$$L_1(t) = (2, 4, 6) + (1, 1, 2)t$$
$$L_2(t) = (-1, 3, 7) + (2, 0, -3)t$$

(a) [8 points] Do the two lines intersect? If so, where?

(b) [7 points] Are the lines perpindicular to one another? Why or why not?

(c) [10 points] Give the equation of the plane that **contains** L_1 and L_2.

(2) Given the curve $\vec{R}(t) =< \cos \pi t, \sin \pi t >$

(a) [5 points] Find $\vec{T}(t)$

(b) [5 points] Find $\vec{N}(t) = \frac{d\vec{T}/dt}{|d\vec{T}/dt|}$.

(c) [8 points] Find the normal and tangential components of the acceleration $\vec{a}$ by one of the following two methods:

i. the way we had discussed in class or in the book
OR

ii. by looking at the size of the projection of $\vec{a}$ onto $\vec{T}$ to get a_t, $a_t = |proj_{\vec{T}}\vec{a}|$, and onto $\vec{N}$, so that $a_n = |proj_{\vec{N}}\vec{a}|$.

(d) [5 points] Graph $\vec{R}(t)$. At $(1, 0)$ (i.e. $t = 0$), draw also the vectors $\vec{T}(0)$ and $\vec{N}(0)$.

(e) [5 points] Does the answer you found in (c) agree with your intuition? Why or why not?

(3) Suppose you have a particle moving along a helix with position given by

$$\sigma(t) =< \cos(2\pi t), \sin(2\pi t), 2\pi t >$$

(a) [5 points] Find $\vec{T}(t)$

(b) [10 points] Find the curvature k

(c) [8 points] Find the arclength of path travelled between $(1, 0, 0)$ and $(1, 0, 2\pi)$.

(c) [7 points] Using the parametric form for a 3D line, give the equation of the **tangent line** to the curve at the point $(1, 0, 2\pi)$. (NOTE: i am not looking for just the tangent vector, but the tan. line!)

(4) A cannon is stationed on an inclined plane at the point (0,0,0). The plane is given by

$$-x - y + 2z = 0 \ .$$

A ball is fired from it at $t = 0$ and it's trajectory follows the path

$$\vec{R}(t) =< t, t, 3t - t^2 > \ ,$$

landing on the plane at time $t = 2$. (See diagram below)

(a) [10 points] Find the velocity and acceleration

(b) [8 points] At what angle does the ball initially leave the plane? (Hint: consider the initial velocity vector $\vec{v}(0)$)
(Note: it is enough to find $\cos(\theta)$)

(c) [7 points] What is the distance between the ball and the plane at time $t = 1$?

(d) [7 points] Does the particle travel in a straight line path between $t = 0$ and $t = 1$? Why or why not?