

My versions of solutions to problems in the first homework set:

#3 (b) Statement: A real number x is irrational if $x + 5$ is irrational.

Writeup #1 Suppose not. Then there is an $x \in \mathbb{Q}$ such that $x + 5$ is irrational. Since $5 \in \mathbb{Q}$ and \mathbb{Q} is closed under addition, $x + 5 \in \mathbb{Q}$ must be true. This contradicts the fact that $x + 5$ is irrational. Hence, x must be irrational.

Writeup #2 (same idea - just written differently) Proof by contradiction: (i.e. show it's negation is false)

Negation: There is some rational number x such that $x + 5$ is irrational.

Proof Negation is False: If $x \in \mathbb{Q}$, then since $5 \in \mathbb{Q}$ and \mathbb{Q} is closed under addition, $x + 5 \in \mathbb{Q}$ must be true. Hence our negation is false.

Since the negation is false, the original statement must be true.

Writeup #3 (Here I'll prove the contrapositive is true, so that the truth of the original statement follows)

Contrapositive: If x is rational, then $x + 5$ is rational.

Proof of Contrapositive: Let $x \in \mathbb{Q}$. Then, since $5 \in \mathbb{Q}$ and \mathbb{Q} is closed under addition, $x + 5 \in \mathbb{Q}$. QED.

Hence the contrapositive is true, so our original statement is true.

(By the way - QED stands for "quod erat demonstrandum", which translates from latin to something like "that which was to be demonstrated". It's just a way to say you've proven what you intended, and is a way to mark the end of a proof.)