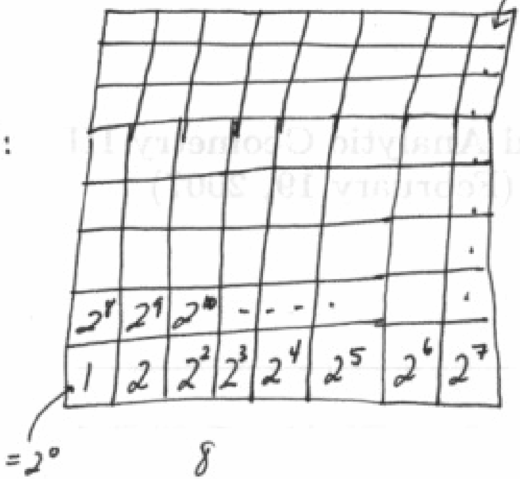
Homework set 2

2.1 :

Problem 8:

2^{63} coins on this spot!



for each spot on the board, the amount of gold increases by 2 times. So by the last one, it should have 2^{63} coins on it!

As Dr. Finotti ~~mentioned~~ mentioned while I was gone, that's enough gold coins that stacking them would actually be more that 1 light-year long!

$2^{62} \approx 4.6 \times 10^{18}$

$2^{63} \approx 9.22 \times 10^{18}$, so the king was to give him more ~~that~~ than 10^{19} gold coins - way more ~~s~~ than he could have possibly had!

1A) - You must take 3 socks to get a matching pair. If the first two aren't the same color (1 black, 1 blue) the next sock must be either black or blue to make a match good!

2 - For two matches, you must take 5 socks. You will get one match from the first 3, leaving you with 1 sock. If you take 2 more socks, you will have a set of 3, 2 of which will be the same color. good!!

- To be sure of a black pair, you must take out 12 socks on the off-chance that the first ten you choose make the 5 blue pairs.

2.2 2) $\varphi = \frac{1+\sqrt{5}}{2}$ The ratios of consecutive Fibonacci numbers approach φ

$\varphi \approx 1.61803$

$\frac{8}{5} \approx 1.6, \frac{13}{8} \approx 1.625, \frac{21}{13} \approx 1.61538$, etc.

#7 (homework set 2):

Pattern for $(F_{n+1})^2 + (F_n)^2$?

let $n=1$: $(F_2)^2 + (F_1)^2 = (1)^2 + (1)^2 = 2 \quad (= F_3\,!)$

$n=2$: $(F_3)^2 + (F_2)^2 = (2)^2 + (1)^2 = 5 \quad (= F_5)$

$n=3$: $(F_4)^2 + (F_3)^2 = (3)^2 + (2)^2 = 13 \quad (= F_7)$

$n=4$: $(F_5)^2 + (F_4)^2 = (5)^2 + (3)^2 = 34 \quad (= F_9)$

so we have

$$F_2^2 + F_1^2 = F_{3=1+2}$$

$$F_3^2 + F_2^2 = F_{5=3+2}$$

$$F_4^2 + F_3^2 = F_{7=4+3}$$

$$F_5^2 + F_4^2 = F_{9=5+4}$$

$$\vdots$$

seems like the pattern is

$$(F_{n+1})^2 + (F_n)^2 = F_{(n+1)+n} = F_{2n+1}.$$

17) $43 = 34 + x$

$43 = 34 + 9$

☆ $43 = 34 + 8 + 1$

$609 = 377 + x$

$609 = 377 + 232$

$609 = 377 + 144 + 88$

$609 = 377 + 144 + 55 + 33$

$609 = 377 + 144 + 55 + 21 + 12$

$609 = 377 + 144 + 55 + 21 + 8 + 4$

☆ $609 = 377 + 144 + 55 + 21 + 8 + 3 + 1$

$90 = 89 + x$

☆ $90 = 89 + 1$

$2000 = 1597 + x$

$x = 403$

$403 = 377 + x$

$x = 26$

$26 = 21 + x$

$x = 5$

$2000 = \cancel{403}\ 1597 + 377 + 21 + 5$

☆ $2000 = 1597 + 403 = 1597 + 377 + 26$

$= 1597 + 377 + 21 + 5$