

2 ✓ There is only one honest politician, because at any given time s/he could be paired with another politician who would have to be crooked, because at least one in a pair must be crooked. You can't have two honest politicians in a pair so there can't be more than one honest politician. good!

2 ✓ There would be 105 lines connecting 15 dots around a circle. The first dot would have 14 lines. The second would have 13 because you can't count a line twice. IF you add $14+13+12$ and so on to one you get 105. good.

2 ✓ Fill the 9-gal pail and pour from it into the 5-gal pail. Throw the 5-gal's water out and pour the remaining 4 gals in the 9 into the 5. Re-fill the 9 from the source and carefully fill up the 5 gal pail from the 9-gal pail leaving 8 gals. Throw the water from the 5 out and refill it from the 9 leaving 3 gals in the 9-gal pail. Throw the water in the 5-gal pail out and pour the 3 gals from the 9-gal pail into the 5-gal pail. Re-fill the 9 from the source and fill the 5 up from the 9. You now have exactly 7 gals of water in the 9-gal pail.

Problem 4, HW set 1:

There are several ways to do this one!

1. You could list them all out... tedious!
2. One idea I had: Make 2 groups, called group A and group B.
In group A, put all the odds from 1-100, then put 2 times all the odds in group B.
Now put $2 \times (2 \times \text{odds})$ in group A, and $2 \times (2 \times (2 \times \text{odds}))$ in B. If you continue in this way until all the numbers are accounted for, you ensure that there is no one in a group that has 2 times the amount of someone else in the same group.

$$A = \{ \text{odds}, 2^2 \times \text{odds}, 2^{\cancel{4} \leftarrow 4} \times \text{odds}, 2^6 \times \text{odds} \}$$

$$B = \{ 2 \times \text{odds}, 2^3 \times \text{odds}, 2^5 \times \text{odds} \}$$

(up to 100 only!
for all of these...)

$$A = \{ 1, 3, 5, 7, 9, 11, 13, \dots, 99; 4, 12, 20, 28, 36, 44, \dots, 100; 16, 48, 80; 64 \}$$

$$B = \{ 2, 6, 10, 14, 18, \dots, 98; 8, 24, 40, 56, 72, 88; 32, 96 \}$$

3. Another idea someone in class had:

basically, follow the patterns

This way we always make sure that 2 times any number is in the other group!

$$\left\{ \begin{array}{l} A: n \rightarrow 2n-1 \\ B: 2n \rightarrow 4n-1 \\ A: 4n \rightarrow 8n-1 \end{array} \right. \quad \begin{array}{l} \leftarrow \text{this is always 1} \\ \text{less than } 2 \times \text{start \#} \end{array}$$

So if we let $n=1$:

$$A \quad 1$$

$$B \quad 2 \rightarrow 3$$

$$A \quad 4 \rightarrow 7$$

$$B \quad 8 \rightarrow 15$$

$$A \quad 16 \rightarrow 31$$

$$B \quad 32 \rightarrow 63$$

$$A \quad 64 \rightarrow 100$$

$$\Rightarrow A = \{1, 4, 5, 6, 7, 16, 17, 18, 19,$$

$$20, 21, 22, 23, 24, 25,$$

$$26, 27, 28, 29, 30, 31,$$

$$64, 65, 66, \dots, 100\}$$

$$B = \{2, 3, 8, 9, 10, 11, 12, 13,$$

$$14, 15, 32, 33, \dots, 63\}$$

* if B contains the numbers $2n$ up to $4n-1$, then 2 times those are all between $4n$ and $8n-2$ which we've put in A!