

2. Since $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$

the closest is 1.62.

3. $\frac{5}{3} = 1.6\bar{6}$

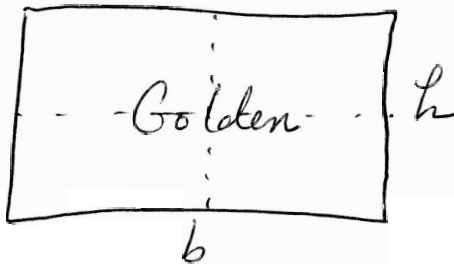
$\frac{14}{11} = 1.2727\dots$

$\frac{11}{8.5} = 1.2941$

$\frac{17}{11} = 1.54\bar{5}$

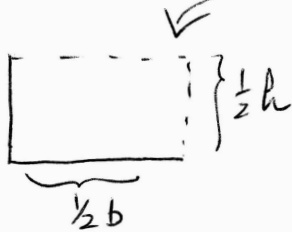
the 3x5 card is closest.

9.



$\Rightarrow \underline{\underline{b/h = \varphi}}$

if I fold it

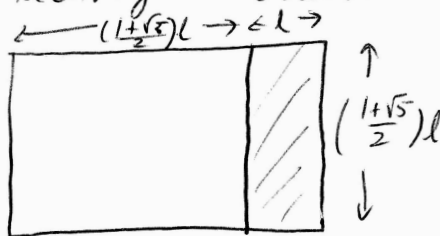


$\Rightarrow \frac{1/2 b}{1/2 h} = \frac{b}{h} = \varphi$

↑ still the ratio of the side lengths is the Golden ratio!

So it's still a Golden rectangle.

12. Yes, the new rectangle is Golden because:



So the new rectangle has sides: $(\frac{1+\sqrt{5}}{2})l + l = \text{base}$
 $(\frac{3+\sqrt{5}}{2})l$

and height = $(\frac{1+\sqrt{5}}{2})l$.

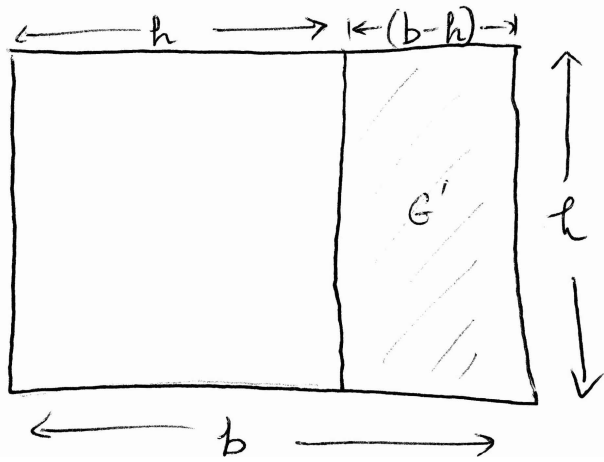
$$\begin{aligned}
 \text{So } \frac{\text{base}}{\text{height}} &= \frac{\left(\frac{3+\sqrt{5}}{2}\right)l}{\left(\frac{1+\sqrt{5}}{2}\right)l} = \frac{(3+\sqrt{5})}{(1+\sqrt{5})} \\
 &= \frac{(3+\sqrt{5})(1-\sqrt{5})}{(1+\sqrt{5})(1-\sqrt{5})} = \frac{3-2\sqrt{5}-5}{1-5} \\
 &= \frac{-2-2\sqrt{5}}{-4} = \frac{-2(1+\sqrt{5})}{-4} \\
 &= \frac{1+\sqrt{5}}{2} = \varphi!
 \end{aligned}$$

So for the bigger rectangles, the ratio of the side lengths is still the Golden ~~ratio~~ ratio, so it's a Golden rectangle!

Since it didn't matter what size the original rectangle was, I could do this again + get another larger Golden rectangle.

13. If you don't start with a golden rectangle, you cannot end up with a golden rectangle. If you did, you could keep removing largest squares until you got back down to the original rectangle. By the argument in the book, this small rectangle would also have to be golden but it isn't!

16.



$$\frac{b}{h} = \varphi$$

$$\text{Area of } G' = h(b-h)$$

$$\text{Area of } G = hb$$

so

$$\frac{\text{Area of } G}{\text{Area of } G'} = \frac{hb}{h(b-h)} = \frac{b}{b-h} \left(\frac{b}{h}\right)$$

$$= \frac{1}{1 - \frac{h}{b}} = \frac{1}{1 - \frac{1}{\varphi}}$$

So it doesn't matter what b and h are, the ratio of the areas will always equal $\frac{1}{1 - \frac{1}{\varphi}}$.