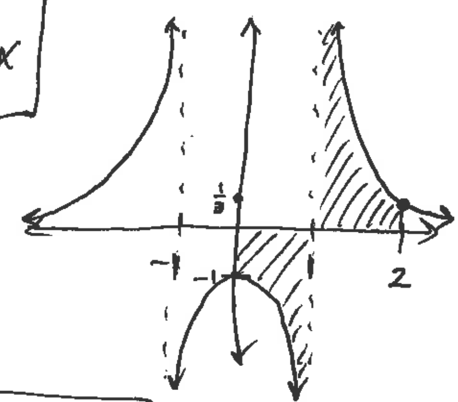
Correcting in class example.

Graph:



Ex: $\displaystyle\int_0^2 \frac{1}{x^2-1}\,dx = \int_0^2 \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}\,dx$

$$= -\frac{1}{2}\int_0^2 \frac{1}{x+1}\,dx + \frac{1}{2}\int_0^2 \frac{1}{x-1}\,dx$$

$$= -\frac{1}{2}\ln|x+1|\Big|_0^2 + \frac{1}{2}\left[\int_0^1 \frac{1}{x-1}\,dx + \int_1^2 \frac{1}{x-1}\,dx\right]$$

$$= -\frac{1}{2}\left(\ln|3|\right) + \frac{1}{2}\left[\lim_{R\to 1^-}\int_0^R \frac{1}{x-1}\,dx + \lim_{R\to 1^+}\int_R^2 \frac{1}{x-1}\,dx\right]$$

$$= -\frac{1}{2}\left(\ln|3|\right) + \frac{1}{2}\left[\lim_{R\to 1^-}\left(\ln|x-1|\Big|_0^R\right) + \lim_{R\to 1^+}\left(\ln|x-1|\Big|_R^2\right)\right]$$

$$= -\frac{1}{2}\ln|3| + \frac{1}{2}\left[\lim_{R\to 1^-}\left(\ln|R-1|\right) + \lim_{R\to 1^+}\left(-\ln|R-1|\right)\right]$$

$$= -\frac{1}{2}\ln|3| + \frac{1}{2}\left[\underbrace{\lim_{R\to 1^-}\left(\ln|R-1|\right)} - \underbrace{\lim_{R\to 1^+}\left(\ln|R-1|\right)}\right]$$

these limits are actually equivalent because of the absolute value bars!

$$\lim_{R\to 1^-}\ln|R-1| = \lim_{R\to 1^+}\ln|R-1| = \lim_{\underline{R\to 1}}\ln|R-1|$$

so their difference is <u>zero</u>.

$$= \underline{-\frac{1}{2}\ln|3|} + 0$$

* This integral <u>converges</u>.