Q: Does it converge or diverge?

Ex: $\displaystyle\int_5^\infty \frac{dx}{x^3-4} \leq \int_5^\infty \frac{dx}{?}$

like $\frac{1}{x^3}$ for large x, so suspect converges

need a denominator here that is smaller than x^3-4 on $[5,\infty)$, so that the quotient is larger than $\frac{1}{x^3-4}$.

on $[5,\infty)$,

we have $x^3 \geq 5^3$

$\Rightarrow \quad \frac{1}{2}x^3 \geq \frac{1}{2}\cdot 5^3 \Rightarrow -\frac{1}{2}x^3 \leq -\frac{1}{2}5^3 < -4$, so $-4 > -\frac{1}{2}x^3$

$x^3-4 = x^3+(-4) > x^3+(-\frac{1}{2}x^3) = \frac{1}{2}x^3$

$\Rightarrow$ on $[5,\infty)$ $x^3-4 > \frac{1}{2}x^3$.

So $\displaystyle\int_5^\infty \frac{dx}{x^3-4} < \int_5^\infty \frac{dx}{\frac{1}{2}x^3} = 2\int_5^\infty \frac{dx}{x^3}$ ← converges

So $\displaystyle\int_5^\infty \frac{dx}{x^3-4}$ converges as well.