Homework Set # 5 – Math 435

1. Consider the Fourier **sine** series of each of the following functions. Do not compute the coefficients, but use the pointwise convergence theorem to discuss the convergence of each of the series. Explain what you would expect to see if you plotted a partial sum approximation via F.S. of each function, paying attention to what would happen at the endpoints.

   (a) $f(x) = x^3$ on $(0, l)$

   (b) $f(x) = lx - x^2$ on $(0, l)$

   (c) $f(x) = \frac{1}{x^2}$ on $(0, l)$.

2. Use the pointwise convergence theorem to explain why it may be that the Fourier series of a given function could converge even though the Fourier series of it's derivative might not.

3. Let

$$\phi(x) = \begin{cases} 1 - (1-x)^2 & \text{for } 0 \leq x < 1 \\ (1-x)^2 & \text{for } 1 \leq x \leq 2 \end{cases}$$

   (a) Find the fourier sine series for $\phi(x)$ over $(0, 2)$.

   (b) Does the series converge pointwise to $\phi(x)$?

   (c) Explain what the Gibbs phenomenon tells you about the behavior of the partial sums of the Fourier Series for ϕ near $x = 1$. [Note: you might find it useful to look at partial sum approximations via a computer so that you can see (b) and (c) "in action"]

4. Following the steps below solve the fourth order BVP $u_{xxxx} = u_t$ if $u(0,t) = 0$, $u(3,t) = 0$, $u_{xx}(0,t) = 0$, and $u_{xx}(3,t) = 0$, and explain how to satisfy an initial condition $u(x, 0) = \phi(x)$.

   (a) Using the method we discussed in class Friday, show that all the eigenvalues of the operator $L(X) = X''''$ are positive.

   (b) Use seperation of variables and the information you obtained in (a) to find the general solution to the BVP.

   (c) Show using the method discussed in class on Thursday that all the eigenfunctions of L given the boundary conditions above are orthogonal to one another, and explain how to apply an intial condition to the solution of the BVP you found in (b).

5. Solve the equation $u_t = u_{xx}$ in $(0, 2)$ with $u_x(0,t) = 1$ and $u_x(2,t) = t$ for all $t > 0$, assuming that $u(x, 0) = 0$. Hint: write u and it's derivatives in terms of Fourier cosine series and proceed as in class on Friday.