Homework Set # 1 – Math 435

1. Determine whether or not the following functions are solutions to the given PDE.
   (a) $u_x - 3u_y = 0$, $u(x,y) = \cos(y+3x)$
   (b) $u_x - 3u_y = 0$, $u(x,y) = 9x^2 + 6xy + y^2$
   (c) $u_{xx} + u_{yy} = 0$, $u(x,y) = x^2 + y^2$
   (d) $u_{xx} - 2u_t = 0$, $u(t,x) = e^t[2e^{\sqrt{2}x} + 3e^{-\sqrt{2}x}]$

2. For the following PDE, determine whether or not they are linear, homogenous, and give their order.
   (a) $u_x + xu_y = 0$
   (b) $u_x + uu_y = 0$
   (c) $u_x + u_y + 1 = 0$
   (d) $u_x + (u_y)^2 = 0$

3. Find the general solution to the PDE $u_{yy} - u = 0$.

4. Suppose you have a linear homogeneous PDE $L(u) = 0$. Suppose that $u_1, u_2, \ldots, u_n$ are all solutions to the PDE. Show that any linear combination of those solutions is itself a solution to the PDE. Show that this is not true if the PDE is inhomogeneous. (Note: This is the superposition principle for PDE.)

5. Verify that $u(x,y) = f(x)g(y)$ is a solution of the PDE $uu_{xy} = u_x u_y$ for all pairs of (differentiable) functions f and g of one variable.(Strauss Problem 11, Section 1.1)

6. The following parts refer to the PDE $2u_t + 3u_x = 0$
   (a) Find the general solution to the PDE. Check that it does in fact solve the PDE.
   (b) If you add the condition that $u(x,0) = cos(x)$, what is $u(x,t)$?
   (c) Sketch the solution $u(x,t)$ for $t = 0, 1, 2$.
   (d) What is the speed of propagation for the transport equation?

7. The following parts refer to the PDE $e^x u_x + u_y = 0$.
   (a) Find the characteristic curves for this PDE and sketch at least 4 of them in the same plane.
   (b) Find the general solution to the PDE.
   (c) If $u(0,y) = (y-2)^2$, find $u(x,y)$.
   (d) Sketch $u(x,y)$ for $y = 0, 1, 2$. Note that we no longer have the same behavior for the solutions "over time" as we had in the constant coefficient case.