3. There are animals that find their food by tracking the scent. They move toward the source of the food by moving in the direction of the greatest increase of scent strength. Suppose the scent strength for a given food source is modeled by

$$s(x, y) = \frac{20}{1 + x^2 + y^2} = 20(1+x^2+y^2)^{-1}$$

and is measured in "olfactometers" (olf's for short).

(a) [8 points] At the point $(0, 2)$, in what direction will the animal move to track it's food?

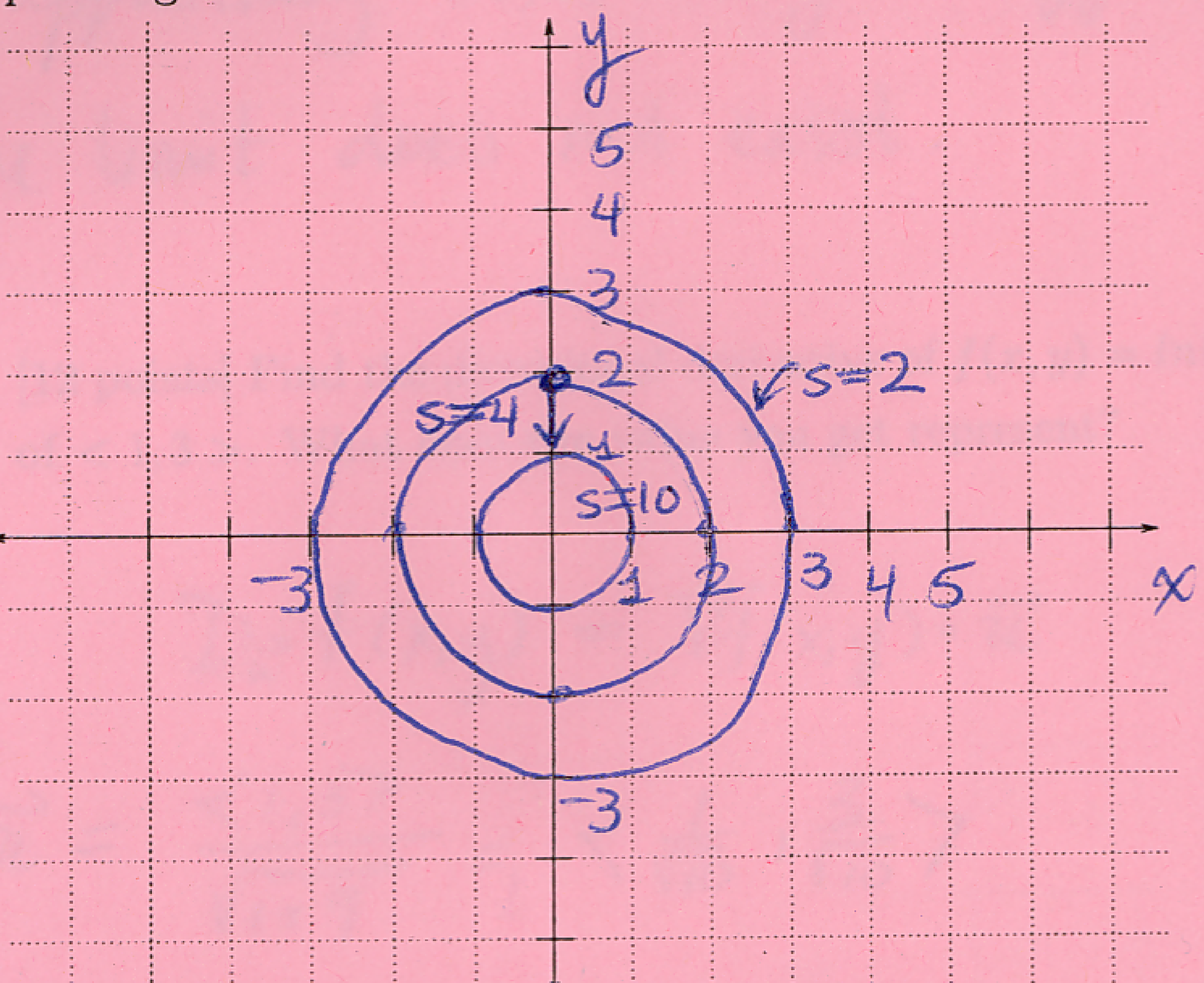
in the direction of $\vec{\nabla} s(x,y)$ at $(0,2)$:

$$\vec{\nabla} s(x,y) = \left\langle -20(x^2+y^2+1)^{-2}(2x),\ -20(1+x^2+y^2)^{-2}(2y) \right\rangle$$

$$\vec{\nabla} s(0,2) = \left\langle 0,\ \frac{-80}{5^2} \right\rangle = \left\langle 0, -\frac{80}{25} \right\rangle$$

$\Rightarrow$ in the negative y-direction.

(b) [8 points] Sketch and label at least 3 level curves of $s(x, y)$, including the one corresponding to $s = 4$. Make sure to label axes and indicate your choice of scale.



$s=4$:

$$4 = \frac{20}{1+x^2+y^2}$$

$$\Rightarrow 1+x^2+y^2 = 5 \Rightarrow \boxed{x^2+y^2=4}$$

$S=10$:

$$10 = \frac{20}{1+x^2+y^2} \Rightarrow 1+x^2+y^2 = 2$$

$$x^2+y^2=1$$

$S=2$:

$$2 = \frac{20}{1+x^2+y^2} \Rightarrow x^2+y^2=9$$

(c) [8 points] Let t be time measured in minutes. If the animal is moving along the path $x(t) = 2 - t$, $y(t) = \sqrt{t}$, how fast is the scent strength changing with respect to time when the bug is at $(x, y) = (1, 1)$?

i.e. what is $\frac{ds}{dt}$?

$$\frac{\partial s}{\partial x} = \frac{-40x}{(1+x^2+y^2)^2} \xrightarrow{@(1,1)} \frac{-40}{9}$$

$$\frac{\partial s}{\partial y} = \frac{-40y}{(1+x^2+y^2)^2} \xrightarrow{@(1,1)} \frac{-40}{9}$$

when $(x,y)=(1,1)$

$\Rightarrow x = 2-t = 1 \Rightarrow t=1$

$y = \sqrt{t} = 1 \Rightarrow t=1$

$$\frac{ds}{dt} = \frac{\partial s}{\partial x}\frac{dx}{dt} + \frac{\partial s}{\partial y}\frac{dy}{dt}$$

$$= \left(\frac{-40}{9}\right)(-1) + \left(\frac{-40}{9}\right)\left(\frac{1}{2}\right)$$

$$= \frac{40}{9} - \frac{20}{9} = \frac{20}{9}\ \frac{\text{olfs}}{\text{min}}$$

$$\frac{dx}{dt} = -1$$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{t}} \quad @\, t=1 \Rightarrow \frac{dy}{dt} = \frac{1}{2}$$