4. (a) [10 points] Does the following limit exist? Explain.

$$\lim_{(x,y)\to(0,0)} \frac{x}{x+y}$$

approaching $(0,0)$
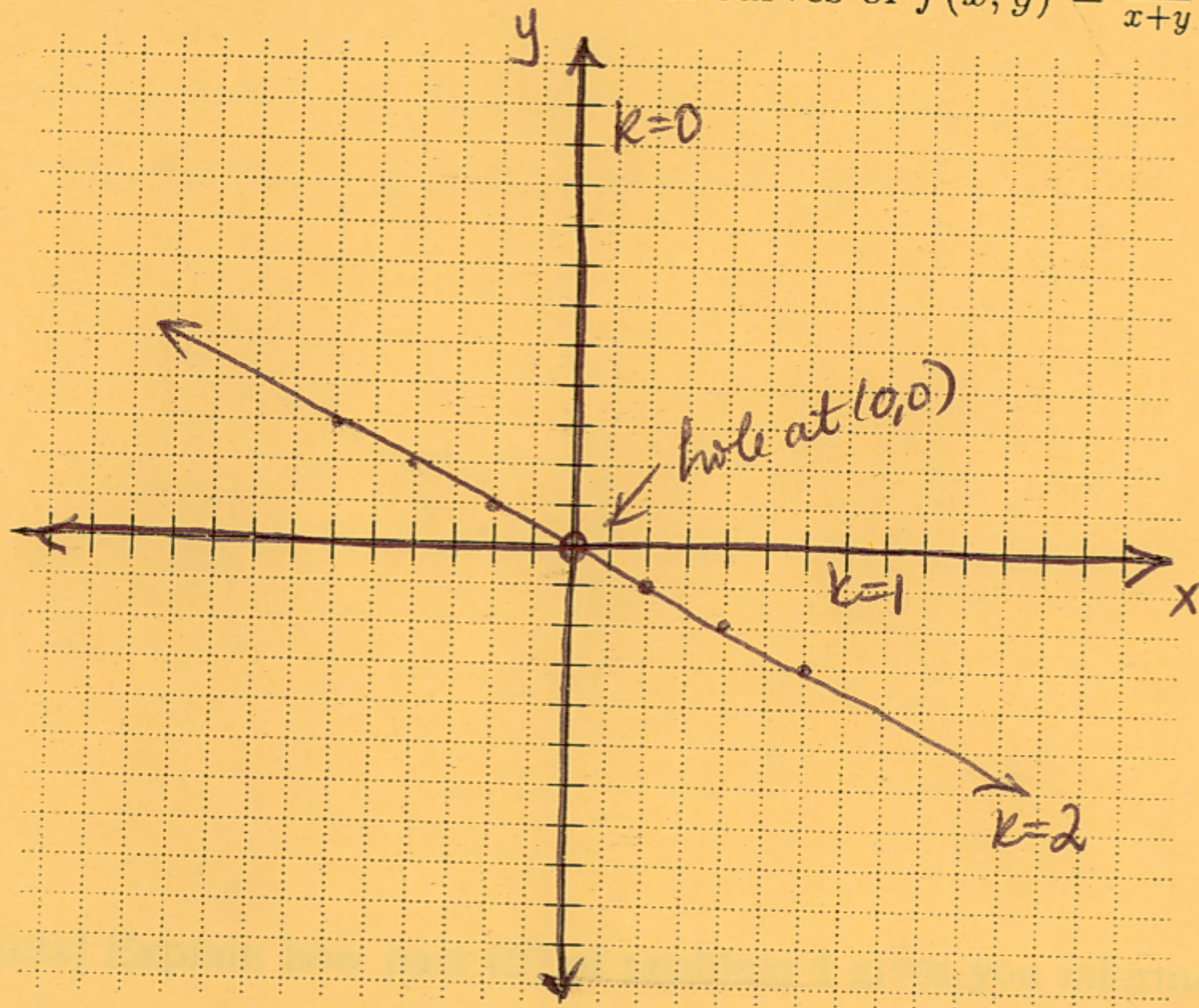along the line $x=0$: $\lim_{(0,y)\to(0,0)} \frac{0}{y} = \lim_{y\to 0} 0 = 0$

approaching $(0,0)$
along the line $y=0$: $\lim_{(x,0)\to(0,0)} \frac{x}{x} = \lim_{x\to 0} 1 = 1$

Since we get two different values for the limit by approaching $(0,0)$ from two different paths, the limit does not exist.

(b) [6 points] Sketch at least 3 level curves of $f(x,y) = \frac{x}{x+y}$

hole at $(0,0)$

$z = k = 0$:

$$0 = \frac{x}{x+y} \Rightarrow \underline{0 = x}$$

$z = k = 1$:

$$1 = \frac{x}{x+y} \Rightarrow x = x+y$$
$$\Rightarrow \underline{y = 0}$$

$z = k = 2$:

$$2 = \frac{x}{x+y} \Rightarrow 2x + 2y = x$$
$$2y = -x$$
$$\underline{y = -\tfrac{1}{2}x}$$