2. Suppose you are hiking on a hill whose surface is described by $f(x,y) = 9 - x^2 - y^2$, and you are at the point on the hill corresponding to $(x,y) = (3,0)$.

(a) [8 points] What slope would you encounter if you walk directly in the x-direction from $(3,0,0)$?

Slope of f in x-direction at $(3,0,0) = f_x(3,0) = \boxed{-6}$

because $f_x(x,y) = -2x$

(b) [8 points] What slope would you encouter if you walk in the direction of $\langle 1,2 \rangle$ from $(3,0,0)$?

$$\vec{u} = \frac{\langle 1,2 \rangle}{|\langle 1,2 \rangle|} = \frac{\langle 1,2 \rangle}{\sqrt{5}} = \langle \tfrac{1}{\sqrt{5}}, \tfrac{2}{\sqrt{5}} \rangle$$

$$D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u} = \langle -2x, -2y \rangle \cdot \langle \tfrac{1}{\sqrt{5}}, \tfrac{2}{\sqrt{5}} \rangle$$

@ $(3,0,0)$:

Slope in direction of $\langle 1,2 \rangle$ at $(3,0,0)$ $= D_{\vec{u}} f(3,0,0) = \langle -6, 0 \rangle \cdot \langle \tfrac{1}{\sqrt{5}}, \tfrac{2}{\sqrt{5}} \rangle = \boxed{-\tfrac{6}{\sqrt{5}}}$

(c) [10 points] In what direction should you go from $(3,0,0)$ if you want to hike up the hill the fastest? (i.e. - in what direction do you go to encouter the greatest incline in the hill?) What is the slope you would encounter moving in this direction?

the gradient always points in the direction of the greatest slope of f at any point:

direction of greatest increase $= \vec{\nabla} f(3,0) = \boxed{\langle -6, 0 \rangle}$
@ $(3,0)$

the slope encountered in the direction of $\vec{\nabla} f$ is $|\vec{\nabla} f| = \boxed{6}$