

x: Solve

$$3t^2 y'' + 11ty' - 3y = t$$

Solution-

$$(1) 3t^2 y'' + 11ty' - 3y = 0$$

$$\text{try } y = t^r$$

$$\Rightarrow 3t^r r(r-1) + 11t^r(r) - 3t^r = 0$$

$$t^r (3r(r-1) + 11r - 3) = 0$$

$$3r^2 + 8r - 3 = 0$$

$$r = \frac{-8 \pm \sqrt{64 + 36}}{6} = \underline{\underline{-3, \frac{1}{3}}}$$

$$y_h = c_1 t^{-3} + c_2 t^{1/3}$$

(3) gen sol'n

$$y = \frac{1}{8}t + c_1 t^{-3} + c_2 t^{1/3}$$

$$(2) \text{ let } y_p = v_1 t^{-3} + v_2 t^{1/3}$$

$$\text{need } v_1' t^{-3} + v_2' t^{1/3} = 0$$

$$\text{and } v_1' (-3t^{-4}) + v_2' \left(\frac{1}{3}t^{-2/3}\right) = \frac{t}{(3t^2)} \leftarrow \text{mistake was here!}$$

Multiplying first by 3 and second by $\frac{t}{3}$ gives:

$$3v_1' t^{-3} + 3v_2' t^{1/3} = 0$$

$$+ \left(-3v_1' t^{-3} + \frac{1}{3}v_2' t^{1/3} = 3\right)$$

$$\frac{10}{3} v_2' t^{1/3} = 3 \Rightarrow v_2' = \frac{1}{10} t^{-1/3} \Rightarrow \boxed{v_2 = \frac{3}{20} t^{2/3}}$$

$$\Rightarrow v_1' = -v_2' t^{10/3} = -\frac{1}{10} t^3 \Rightarrow \boxed{v_1 = -\frac{1}{40} t^4}$$

$$y_p = \cancel{\frac{1}{40}t} - \frac{1}{40}t + \frac{3}{20}t = \frac{5}{40}t = \boxed{\frac{1}{8}t}$$