— SECTION 4.5 —

#19. $y'' - 3y' + 2y = e^x \sin x$

guess: $y_p = Ae^x \sin x + Be^x \cos x$

Check homog. sol'n:

$y'' - 3y' + 2y = 0$

try $y = e^{rx} \Rightarrow r^2 - 3r + 2 = 0$

$(r-1)(r-2) = 0$

$r = 1, 2.$

$y_h = c_1 e^{x} + c_2 e^{2x}$

So $y_p = Ae^x \sin x + Be^x \cos x$ is good since no term of y_p is a sol'n to the homog. problem.

$\Rightarrow y_p' = Ae^x \sin x + Ae^x \cos x + Be^x \cos x + -Be^x \sin x$

$y_p' = (A-B)e^x \sin x + (A+B)e^x \cos x$

$y_p'' = (A - B - (A+B))\, e^x \sin x + (A+B+A-B)\, e^x \cos x$

$y_p'' = -2Be^x \sin x + 2Ae^x \cos x$

$\Rightarrow y_p'' - 3y_p' + 2y_p = (-2B - 3A + 3B + 2A)e^x \sin x + (2A - 3A - 3B + 2B)e^x \cos x$

$= (B-A)e^x \sin x + (-A-B)e^x \cos x \overset{\text{want}}{=} e^x \sin x$

$\Rightarrow B - A = 1$

$+ \; (-A - B = 0)$

$\Rightarrow -2A = 1$

$A = -\frac{1}{2}.$ so since $B = -A \Rightarrow B = \frac{1}{2}.$

$\left[y_p = -\frac{1}{2} e^x \sin x + \frac{1}{2} e^x \cos x \right]$

— Section 4.5 —

#27: $y'' - y' - 2y = \cos x - \sin(2x)$

① homog: $y'' - y' - 2y = 0$

try $y = e^{rt} \Rightarrow r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$r = 2, -1$

$$\boxed{y_h = c_1 e^{2t} + c_2 e^{-t}}$$

② nonhomog:

guess: $y_p = \underbrace{A\cos(x) + B\sin(x)}_{\text{because of } \cos x \text{ on RHS}} + \underbrace{C\cos(2x) + D\sin(2x)}_{\text{because of } \sin(2x) \text{ on RHS.}}$

$\Rightarrow$

$y_p' = -A\sin(x) + B\cos(x) - 2C\sin(2x) + 2D\cos(2x).$

$y_p'' = -A\cos(x) - B\sin(x) - 4C\cos(2x) - 4D\sin(2x)$

So $y_p'' - y_p' - 2y_p = (-3A - B)\cos(x) + (-3B + A)\sin(x)$

$+ (-6C - 2D)\cos(2x) + (-6D + 2C)\sin(2x)$

$\overset{\text{want}}{=} \cos x - \sin(2x)$

So $\begin{cases} -3A - B = 1 \\ -3B + A = 0 \end{cases}$ and $\begin{cases} -6C - 2D = 0 \\ -6D + 2C = -1 \end{cases}$

$\Rightarrow B = -\frac{1}{10}, \; A = \frac{3}{10}$

$-20D = -3$

$D = 3/20, \; C = -1/20.$

So $\boxed{y_p = \frac{-3}{10}\cos(x) - \frac{1}{10}\sin(x) - \frac{1}{20}\cos(2x) + \frac{3}{20}\sin(2x)}$

and the general solution is

$$\boxed{y = c_1 e^{2t} + c_2 e^{-t} + \frac{-3}{10}\cos(x) - \frac{1}{10}\sin(x) - \frac{1}{20}\cos(2x) + \frac{3}{20}\sin(2x)}$$

if $y(0) = \frac{-7}{20} = c_1 + c_2 + \frac{-3}{10} - \frac{1}{20}$

$$\Rightarrow c_1 + c_2 = 0$$

and $y'(0) = 2c_1 - c_2 - \frac{1}{10} + \frac{6}{20} = \frac{1}{5}$

$$\Rightarrow 2c_1 - c_2 = 0$$

So adding gives:

$$3c_1 = 0 \Rightarrow \boxed{c_1 = 0}$$

$$\Rightarrow \boxed{c_2 = 0}$$