

#1 - section 3.4:  $y(0) = 0$

$y(t) = 1000$

model: if $v(t)$ = velocity of object, and $y(t)$ = height, then $v = y'$

and: mass · acceleration = net force

$$\Rightarrow m y'' = mg - by'$$

or $\boxed{m v' = mg - bv}$

$$\Rightarrow 5v' = 5(9.8) - 50v \Rightarrow v' = 9.8 - 10v$$

$$\Rightarrow v' + 10v = 9.8 \Rightarrow \int (v' + 10v) dt = \int 9.8 dt = 9.8t + C$$

$$\Rightarrow (e^{10t} v)' = 9.8 e^{10t} \Rightarrow e^{10t} v = 0.98 e^{10t} + C$$

$$\boxed{v = 0.98 + C e^{-10t}}. \text{ Since } v(0) = 0$$

$$\Rightarrow 0.98 + C = 0 \Rightarrow C = -0.98$$

$$y' = v(t) = 0.98 - 0.98 e^{-10t}$$

$$\Rightarrow y(t) = 0.98t - 0.098 e^{-10t} + C$$

and since $y(0) = 0 \Rightarrow C = 0.098$, so $\boxed{y(t) = 0.98t - 0.098 e^{-10t} + 0.098}$

Find t so that $y = 1000$:

$$1000 = 0.98t - 0.098 e^{-10t} + 0.098$$

$$\Rightarrow 999.902 = 0.98t - 0.098 e^{-10t}$$

$$\Rightarrow \frac{999.902}{0.98} = t - \frac{1}{10} e^{-10t}$$

for t near 1000, e^{-10t} is negligible,

so we can approximate

$$e^{-10t} \approx 0 \Rightarrow t \approx \frac{999.902}{0.98} \approx 1020 \text{ seconds}$$