Department of Mathematics
The University of Tennessee

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H. Finotti

Math 231:Introduction to Ordinary Differential Equations
Mini-Project: Spread of Staph Infections in Hospitals

Read the synopsis given in pages 310-312 of our text. The final system you will work with is:

$$\frac{dH}{dt} = \frac{\beta_H}{N}(N - C - H)H - (\delta_H + \alpha_H)H \tag{1}$$

$$\frac{dC}{dt} = \frac{\beta_C}{N}(N - C - H)C - (\delta_C + \alpha_C)C \ . \tag{2}$$

We can write this in a bit of a more "friendly" form in terms of the phase plane analysis as:

$$\frac{dH}{dt} = \frac{\beta_H H}{N}[N(1 - \frac{\delta_H + \alpha_H}{\beta_H}) - C - H] \tag{3}$$

$$\frac{dC}{dt} = \frac{\beta_C C}{N}[N(1 - \frac{\delta_c + \alpha_c}{\beta_C}) - C - H] \ . \tag{4}$$

For simplicity of notation, let's let $D_H = N(1 - \frac{\delta_H + \alpha_H}{\beta_H})$ and $D_C = N(1 - \frac{\delta_c + \alpha_c}{\beta_C})$. The dynamics of this system essentially hinges on these two constants as we shall see, and the system is then written as

$$\frac{dH}{dt} = \frac{\beta_H H}{N}[D_H - C - H] \tag{5}$$

$$\frac{dC}{dt} = \frac{\beta_C C}{N}[D_C - C - H] \ . \tag{6}$$

This project is written for three members. One member is responsible for problem 1, one for problem 2, and one for problem 3. The remaining question is to be answered as a group.

1. First, write out the steps required to go from the form of the system of equations given in the first expression of our system above to the form given in the second expression of our system above.

   Next, use the parameter values on page 312 of the text and do a full phase plane analysis of this system by hand. Find any equillibria, list all possible long-term outcomes, and what they would mean for the spread of the different staph strains in the hospital.

Using MATLAB, solve the system for $H(t)$ and $C(t)$ three times, using three different sets of initial values, each set from different regions of your phase plane. Examining your work from problems 1 and 2, what does this mean about whether or not CA-MRSA will overtake HA-MRSA in the hospital?

Suppose a drug was developed that reduced the average duration of both types of MRSA strains by 2 days. Redo the phase plane work and interpretation, and the MATLAB investigation for this new system. Explain the results you get in terms of the change we've made in the disease duration.

2. Now, let's start to investigate how length of hospital stay effects long term outcomes. Reasonably, it would seem that if we could reduce the length of a hospital stay for the CA-MRSA patients, possibly we could reduce the spread of this strain, since it would allow for less time for these patients to come into contact with others (directly or indirectly). First, see if there is any difference in the outcome for the disease if the length of stay for the CA-MRSA patients is reduced to 5 days. Redo the phase plane and comment on the outcome. If the dynamics are not changed, try 4 days, and so on. This gives you an idea about how you could determine a time window in which you would need to somehow effectively remove the patients from the hospital in order to markedly change the outcome long term. If you find a stay time that changes the dynamics, show us this phase plane along with MATLAB computed trajectories for several different intial conditions that illustrate the various long-term outcomes possible. Reduced stay time could possibly be achieved through removal to a special home care situation earlier, or through quarantine within the hospital, or possibly by having a more efficient means of diagnosis, in order to catch the presence of the disease and treat it appropriately sooner.

3. Finally, we can investigate the impact of transmission rates of HA-MRSA and CA-MRSA on the spread of CA-MRSA. Transmission rates could potentially be modified through quarantine again, or through thorough "hygiene" and cleaning practices for the hospital staff and janitors. The question remains as to whether or not such measures would be enough to change the spread of CA-MRSA long term, and if so, how much the transmission rates would need to change by in order to be effective. First investigate whether or not the eventual spread of CA-MRSA would be changed if the transmission rate for CA-MRSA is reduced to the same as that for HA-MRSA? How small would β_C need to be in order to make sure that CA-MRSA does not overtake HA-MRSA in the hospital? Create (at

least one) phase plane(s) to help you answer this question, discuss the possible outcomes indicated by these phase planes, and use MATLAB to find at least three trajectories illustrating interesting long-term outcomes for each of the phase planes.

4. In the project introduction above, I state that the key to the dynamics of this system is really the relationship between the values of D_H and D_C. Looking over your work in the previous problems, what is really the deciding factor for whether or not CA-MRSA will overtake HA-MRSA in the hospital in terms of the relationship between D_H and D_C? Recognizing this key gives other hosptials a very quick and easy way to compute whether or not CA-MRSA will be under control in their own hospitals or not.

Citations: See text page 312