Examples

Example 1 An example of a (nonstiff) system is the system of equations describing the motion of a rigid body without external forces.

$$y_1' = y_2 y_3 \qquad y_1(0) = 0 \tag{1}$$

$$y_2' = y_1 y_3 \qquad y_2(0) = 1 \tag{2}$$

$$y_3' = -0.51 y_1 y_2 \qquad y_3(0) = 1 \tag{3}$$

To simulate this system, create a function called rigid containing the equations. This function should be saved as rigid.m and written in a basic text editor like emacs or notepad.

```
function dy = rigid(t,y)
dy = zeros(3,1); % a column vector, this initializes the vector dy to all zeros
dy(1) = y(2) * y(3);
dy(2) = -y(1) * y(3);
dy(3) = -0.51 * y(1) * y(2);
```

In this example we solve on a time interval [0 12] with an initial condition vector [0 1 1] at time 0. You will need to decide how long a time interval you need to solve over in order to see the long term behavior of the system. Enter the line below directly into the MATLAB prompt. Make sure that MATLAB is currently open to the directory that contains your rigid.m file so that it can read it.

```
[T,Y] = ode45(@rigid,[0 12],[0 1 1]);
```

Plotting the columns of the returned array Y versus T shows the solutions. To do this, enter the line below at the MATLAB prompt after running the previous line.

```
plot(T,Y(:,1),'-',T,Y(:,2),'-.',T,Y(:,3),'.')
```

Save your plots! You will want to include these and your code with your write up. Make sure they are clearly labeled when you turn them in, so that I can see what they represent without having to refer to your code. If you get any error messages while running this, and do not know how to debug it, let me know (especially if there is anything about tolerances being reached).

Example 2 Here is another example - if using ode45 is extremely slow in solving your problem, you may try using ode15s, as your problem may be "stiff" (has regions of very sharp change). An example of a (stiff) system is provided by the van der Pol equations in relaxation oscillation. The limit cycle has portions where the solution components change slowly and the problem is quite stiff, alternating with regions of very sharp change where it is not stiff.

$$y_1' = y_2 \qquad y_1(0) = 2 \tag{4}$$

$$y_2' = 1000(1 - y_1^2)y_2 - y_1 \qquad y_2(0) = 0 \tag{5}$$

To simulate this system, create a function saved as vdp1000.m containing the equations

```
function dy = vdp1000(t,y)
dy = zeros(2,1); % an initializing column vector
dy(1) = y(2);
dy(2) = 1000*(1 - y(1)^2)*y(2) - y(1);
```

For this problem, we will solve on a time interval of [0 3000] with initial condition vector [2 0] at time 0.

```
[T,Y] = ode15s(@vdp1000,[0 3000],[2 0]);
```

Plotting the first column of the returned matrix Y versus T shows the solution

```
plot(T,Y(:,1),'-o')
```