Using the determinant of a matrix to find the cross product of two vectors:

Here I will define what is meant by the notation.

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

for a general matrix (3x3)

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

we write the determinant of A by:

$$\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

which is computed in the following way:

(1) block out the row + column containing a:

$$\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ \cancel{d} & e & f \\ \cancel{g} & h & i \end{vmatrix}$$

and "cross multiply" the remaining entries

$$\begin{matrix} e & f \\ h & i \end{matrix} \rightarrow \underset{①}{ei} - \underset{②}{fh}$$

This gives us our first contribution to the determinant : $a(ei - fh)$

Now do the same with b, but when you "cross multiply" the remaining entries, use the opposite order

$$\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ d & \cancel{e} & f \\ g & \cancel{h} & i \end{vmatrix} \rightarrow \begin{matrix} d & f \\ g & i \end{matrix} \rightarrow \underset{1}{fg} - \underset{2}{di}$$

So the next contribution to the determinant is

$$b(fg - di)$$

Finally repeat for c, doing the "cross-multiplication" in the same order as for a:

$$\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ d & e & \cancel{f} \\ g & h & \cancel{i} \end{vmatrix} \rightarrow \underset{1}{dh} - \underset{2}{eg}$$

$\Rightarrow$ contribution is $c(dh-eg)$

<u>then</u>:

$$\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei-fh) + b(di-fg) + c(dh-eg)$$

So for our example:

by definition of determinant of a matrix

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \vec{i}\,(v_2 w_3 - v_3 w_2) + \vec{j}\,(v_3 w_1 - v_1 w_3) + \vec{k}\,(v_1 w_2 - v_2 w_1)$$

= algebraic definition of cross product!