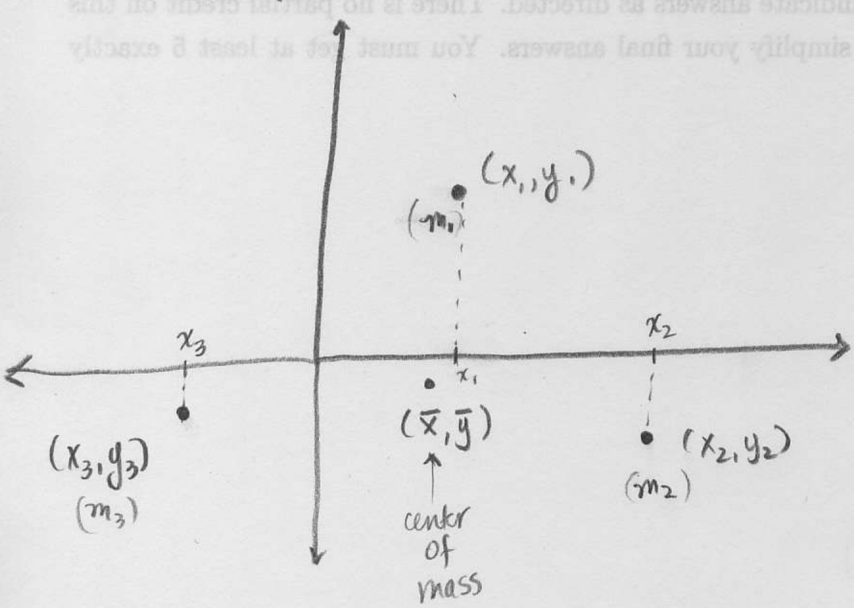
Center of Mass in two dimensions:

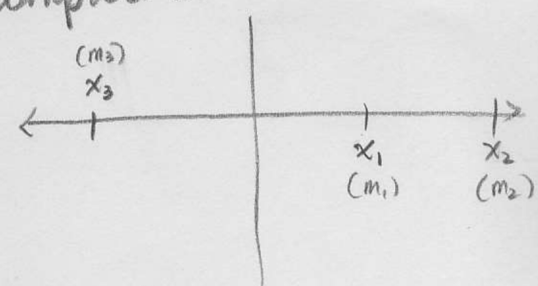
First Key Idea: Two find the center of mass of point masses in a plane or a two-dimensional object with density (constant) δ, YOU CAN FIND the center of mass for each coordinate individually – as though the masses where in a line rather than over a plane!

Example:

(Reduce to linear case in each component)

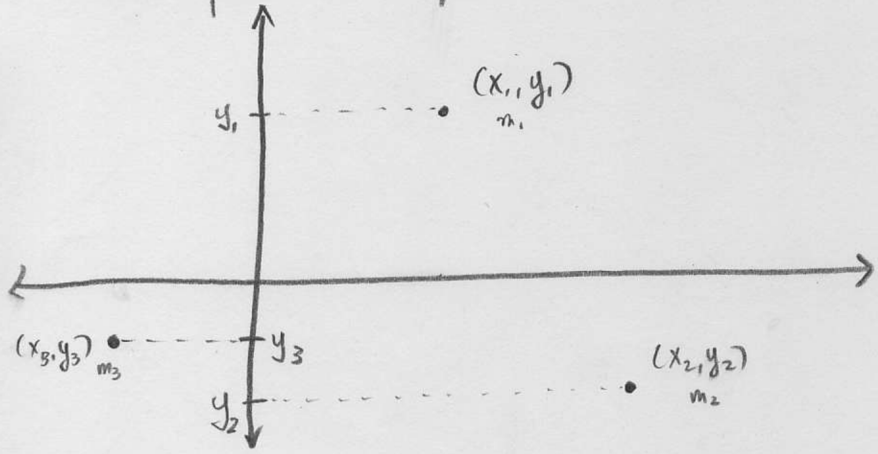


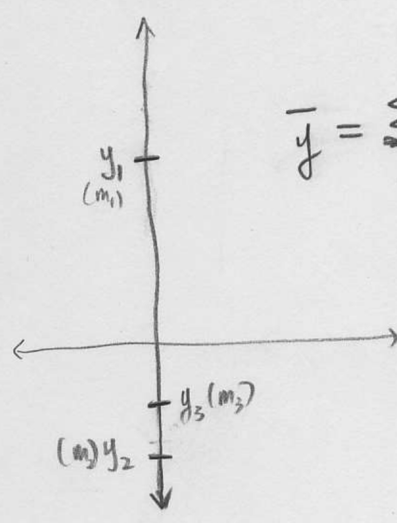
to find $\bar{x}$ basically ignore y-component and compute as usual:



$$\bar{x} = \frac{\sum_{i=1}^{3} m_i x_i}{\sum m_i}$$

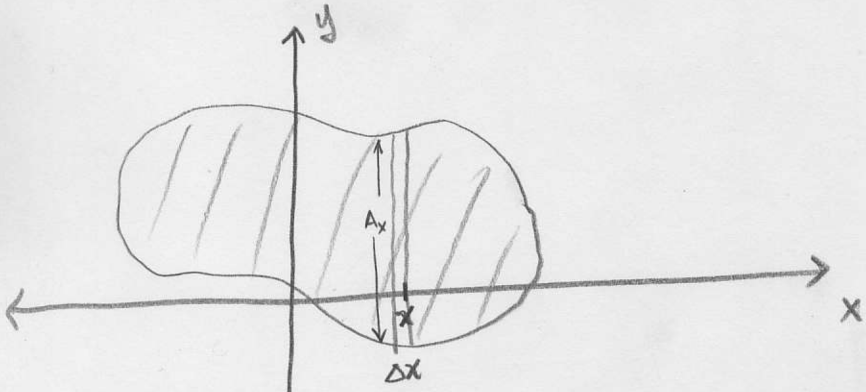
and for $\bar{y}$ – ignore the x-component and compute as before:





$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i}$$

So this is how it's done for discrete point masses in a plane. How about if I have a 2-D object with constant density δ?



follow same ideas:

for $\bar{x}$: • slice at values of x (i.e. vertically)

• the mass of that slice is

mass = area x density $= (A_x(x) \cdot \Delta x) \cdot \delta$

where $A_x(x)$ = height of slice at x.

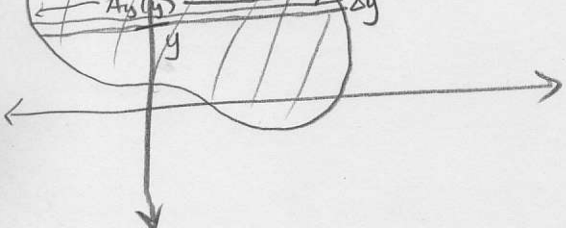
$\Rightarrow$ moment for that slice is $\quad x \cdot A_x(x) \Delta x \cdot \delta$

so $\quad \bar{x} \cong \dfrac{\sum x A_x(x) \Delta x \, \delta}{\sum A_x(x) \Delta x \, \delta} \Rightarrow \quad \bar{x} = \dfrac{\int \delta x A_x(x)\,dx}{\int \delta A_x(x)\,dx}$

[Notice this is like reducing to the 1-D case by putting all of the mass at a fixed value of x (regardless of y-coord) as a point mass on the x-axis at location of the slice!]

Similarily for $\bar{y}$: . slice at a value of y (horizontally)

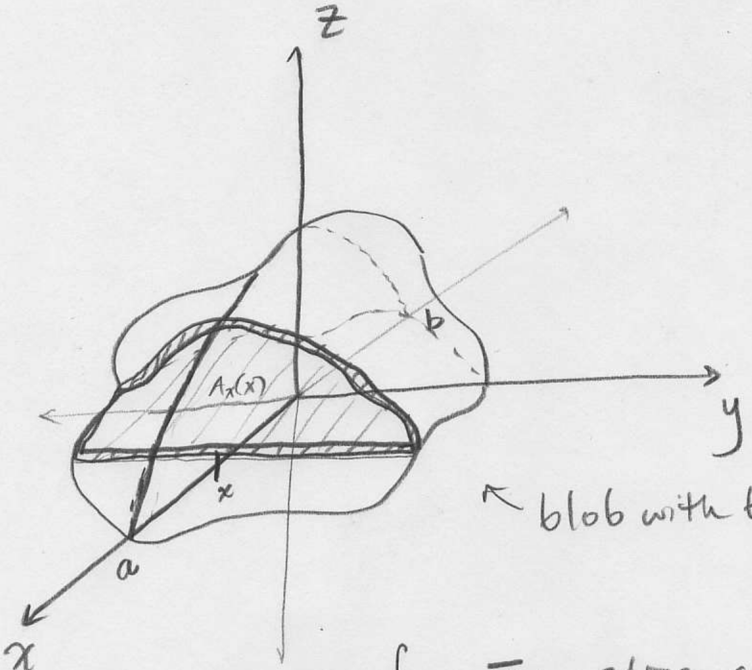
• mass of slice = $\underbrace{\delta}_{\text{density}} \cdot \underbrace{A_y(y) \Delta y}_{\text{area}}$



• moment of slice = $y \cdot \delta A_y(y) \Delta y$

$$\bar{y} = \frac{\int \delta y A_y(y)\,dy}{\int \delta A_y(y)\,dy}$$

3D:



↖ blob with base in x-y plane.

for $\overline{x}$: . slice along a value of x

- If the density is constant $= \delta$,

mass of slice $\cong \underbrace{A_x(x)\Delta x}_{\text{volume}} \cdot \underbrace{\delta}_{\text{density}}$

where $A_x(x) =$ surface area of slice at x!

moment of slice $\cong x \cdot A_x(x)\Delta x \cdot \delta$

$$\Rightarrow \overline{x} = \frac{\int_a^b \delta x A_x(x)\,dx}{\int_a^b \delta A_x(x)\,dx}$$

(Similarly: $\overline{y} = \dfrac{\int \delta y A_y(y)\,dy}{\int \delta A_y(y)\,dy}$, $\overline{z} = \dfrac{\int \delta z A_z(z)\,dz}{\int \delta A_z(z)\,dz}$)

[Again, this is like concentrating all of the mass in a slice at x into a point mass at x on the x-axis - ignoring other coordinates - so we essentially reduce to the 1D case again.]