π Day Problems - March 2010

1. Pheidippides' favorite game is Pin the Tail on the Tau. He practices a lot. He succeeds in pinning the tail onto the τ while blind-folded 93% of the time. If you let Pheidippides play the game ten times, how many times is he most likely to pin the tail on the τ?

2. Nate wants to make 25 pies. He finds recipes for three different types of pies: apple, pumpkin and delicious pecan. In how many ways can Nate make the 25 pies with these three recipes?

3. Katelyn bakes a pie and then marks four points on it. She marks A, B, and C arbitrarily around the edge and O at the center. Show that $\angle AOC = 2 * \angle ABC$.

4. Evaluate
$$\lim_{n\to\infty} \left(1 + \frac{\pi * i}{n}\right)^n.$$

5. Sarah goes for a walk in the Buda hills. She finds a bunch of strange square roots. She likes how the roots smell and decides to try to make a pie out of them, but she isn't sure if it is possible. Solve this equation to help Sarah make her pie:
$$\pi = \sqrt{b + \sqrt{b + \sqrt{b + \sqrt{b + \ldots}}}}$$

6. Let $p = 314,159$. Show that p has a multiple whose digit sum is itself.

7. Tim is looking through the slides of a colloquium talk. He finds an interesting identity and wants to prove it:
$$\sum_{k=0}^{m} \binom{2n - m - 1 + k}{n - k} \binom{m + k}{k} = \binom{2n - 1}{n}$$
Tim is tired of using computers, so he looks for a combinatorial argument. He prepares to solve the problem by eating a piece of pie. Can you find such a combinatorial argument for the truth of the identity?

8. Which is better, π or e? Include justification for each step of your proof.