Final Exam

Name: Grading Guide

Each problem is worth 30 points. Work at least 10 problems; you may work one additional problem as a bonus problem worth 15 points. Be sure to indicate which problem is to be graded as a bonus; if you fail to do so I will grade the first 10 problems you work out of 30 and the eleventh out of 15. If you work all 12 I will only grade the first 11. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested. Use of calculators with symbolic algebraic capability such as the TI-89 is prohibited and will result in a grade of zero on this exam.

1. Use the definition of the integral (NOT FTC) to evaluate

$$\int_0^3 x^3 \mathrm{d}x$$

(Hint: Use the fact that $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$.)

(a) +3 Anything only vaguely similar to generic formula/ No other work

(b) −5 major alg

(c) −5 bounds doesn't slightly

(d) +10 vaguely similar to generic formula, does some additional work

(e) −3 minor arith

(f) −5 minor notational problems

(g) +15 mostly similar to generic form; screws up ∆x

2. Find

(a)

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x \sqrt{\sec \ln \frac{t^2+3}{t}}\,\mathrm{d}t$$

(b)

$$\int_a^b \left(\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{\sec \ln \frac{x^2+3}{x}}\right)\mathrm{d}x$$

(a) -15 on For anything that looks like taking a derivative or antiderivative

(b) -10 does $\sqrt{\sec \ln \frac{x^2+3}{x}}$ for (b)

(c) -5 minor notational problems in an intermediate step

(a) −1 no +C

3. Find the indefinite integral:

6 (a) $\int 5^x dx$
6 (b) $\int -\frac{1}{\sqrt{1-t^2}} dt$
6 (c) $\int x^3 e^x dx$
6 (d) $\int x^2 e^{(x^3)} dx$
6 (e) $\int \cos^2\theta d\theta$

(a) (a) 5^{-2} does (incl 5^t (e) no ln 5
(b) −5 "prob"
(c) −5 takes a ln or a der
(d) −5 use parts (incorrectly)

(b) (a) −5 some trig fn, not an inverse
(b) −4 some inverse trig fn, not $-\sin^{-1}t$ or $\cos^{-1}t$
(c) −2 bad trigid (d) −1 sign error

(c) (a) −2 off by const. factor
(b) −3 sign error in parts formula
(c) −3 barks prob
(d) −5 no parts
(e) −1 other sign error
(f) −3 drops terms
(g) −5 attempts subs
(h) −4 attempts to add integral to other side & solve

(d) (a) −5 attempts parts
(b) −2 barks prob & off by const factor
(c) −5 prob only

(e) (a) −3 barks trigid totally
(b) −5 uses prob; no trigids / pythid only
(c) −2 $\int \cos\theta = \cos\theta$
(d) −3 extra unexplicable junk in antider.
(e) −4 doesn't show work
(f) −2 $\int \cos^2\theta d\theta = \frac{1}{2}\sin\theta$
(g) −1 sign err. in trigid (h) −2 off by const. factor

4. Find the indefinite integral:

$$\int \sqrt{1-x^2}\,dx$$

(a) +8 just gives an inverse trig fn w/ no justification

(b) +5 Attempts prob

(c) −7 doesn't solve for fn in x, or bottes resubs

(d) −3 doesn't simplify $\sin(2\sin^{-1}x)$, or simplifies incorrectly

(e) −7 wrong trigid

(f) −3 bad u-subs

(g) −2 sign error in du

(h) −10 doesn't evaluate the integral −12 after subs, or nonsensical antideriv

(i) −3 doesn't change the variable

5. Find the indefinite integral:

IS pfrac

IS antiders

$$\int \frac{3x^3 + x^2 - 2x + 4}{x^4 + x^2} \mathrm{d}x$$

(a) +12 Just uses prule everywhere

(b) -3ea barks an elem. antider. or leaves it out

(c) -5ea err in the generic decomp.

(d) -15 max -10 no pfrac

(e) -7 major alg-

(f) +5 $\int fg = \int f \int g$

(g) -5 doesn't show work solving for coeffs

(h) -12 tries long division

6. Suppose the velocity of a particle at time t is given by $v(t) = t^3 - 3t^2 + 2t$.

15 (a) Find the displacement of the particle after 10 seconds, assuming it starts at the origin.

15 (b) Find the distance traveled by the particle during the first 10 seconds.

(a) +5 finds $v(10)$ & $a(10)$

(b) −2 minor arith.

(c) −7 splits integral apart at nonsensical pt.

(d) −1 ea sign errs.

(e) −5 splits interval apart at zeros but doesn't take abs. val.

7. Approximate using Simpson's rule with $n = 4$:

$$\int_0^1 x^2 \mathrm{d}x$$

(a) −10 Uses midpoints instead of endpts

(b) −5 Uses wrong b/a for $\frac{b-a}{2n}$

(c) −5 only does n pts

(d) −2 minor arith

(e) −10 no coeffs

(f) −5 coeffs in wrong order

8. Find the area enclosed by $y = x^2$ and $x = y^2$.

(9) — (8) 5 sq units a dr

9. Find the volume of the solid formed by rotating the region bounded by $y = \log_2 x$ and $y = (x-1)^2$ about the y-axis.

(a) $-\frac{18}{20}$ gives a formula for area rather than volume or other non-volume

(b) -5 can misses an antider.

(c) $\rightarrow \log_2 x = \ln x - \ln 2$

(d) -5 a doesn't take an antider by hand

(e) -10 Tries to use washers but doesn't solve for x

(f) -2 on bad isection

(g) -5 Uses disks rather than annuli

(h) -10 Gives mix-n-wrong w/o formula

10. Suppose a chain 500ft in length weighs 3000lbs. Find the work required to lift the chain to the top of a building 500ft high.

(a) -5 extra nonsensical constant factor or added in gravity

(b) -5 uses recip of linear density

(c) -5 Gives units as N?

(d) -10 doesn't take an integral but does some setup

(e) -5 gives units as J or no units

(f) -10 doesn't multiply by distance

11. 16(a) Give the Maclaurin Series for $f(x) = \sin x$

10 (b) Use the alternating series test to determine how many terms are needed to approximate $\sin \frac{\pi}{12}$ accurate to 6 decimals.

16(c) Find the indicated approximation

(a) (a) -2 leaves out ! but gets it in (b) (c)

(b) -5 leaves out Σ

(c) -5 gives series for $\cos x$

(1) (a) -3 uses n in table or does $2n+2$ instead of $2n+3$

(0)

(a) -5 uses $\pm \pi/12$

(b) -5 series doesn't alternate

(0)

-9 Just restates series w/ $\infty = 3$, or similar.

(d) -6 # terms not consistent w/ (b)

12. 10(a) Find the Taylor series expansion of $f(x) = \sin x$ at $a = \pi$

10(b) Use Taylor's inequality ($R_n(x) \leq \frac{M}{(n+1)!}|x-a|^{n+1}$, where $\left|f^{(n+1)}(x)\right| \leq M$) to determine how many terms of the series are needed to approximate $\sin \frac{11\pi}{12}$ accurate to 6 decimals.

10(c) Find the indicated approximation.

(a) (a) – 5 Just gets .965758 & no series

(b) (a) – 7 makes a table but works at C

(c) (a) –10 doesn't show work