

Name: Grading Guide

Each problem is worth the indicated number of points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested. Use of calculators with symbolic algebraic capability such as the TI-89 is prohibited and will result in a grade of zero on this exam.

1. (15 pts) Determine whether the sequence converges or diverges. If it converges, find the limit.

-everything  
treats  
as  
series

3	(a) $\{\cos \pi n\}$	a) -2 diverges why?
4	(b) $\{\sin \pi n\}$	-4 converges 0
4	(c) $\{\frac{2^n}{n^2}\}$	3
4	(d) $\{\frac{(-1)^n}{n^2}\}$	

b) -2 converge to wrong pt  
-4 claims oscillation

c) -1 makes a table comparing  $n^2$  &  $2^n$   
-3 converges to zero via l'hôpital -4 converges to 0  
-2 diverges no l'hôpital no reason  
-2 compares the wrong way -2 compares no proof

d) -2 takes abs. val & claims diverge  
-3  $\Rightarrow 0$  why?  
-3  $\Rightarrow 0$  nonsensical reason  
-4 claims oscillation or other diverge

2. (20 pts) For the function  $f(x) = \sin x$

4 (a) Give the generic form of a Maclaurin series

10 (b) Find the Maclaurin series for  $f(x)$

6 (c) Use the Maclaurin series to approximate  $\sin \frac{\pi}{12}$ , accurate to 5 decimals.



4. (20 pts)

7 (a) Find a power series representation of  $f(x) = \frac{1}{1-x^2}$

7 (b) Use part 4a to find a power series representation of  $g(x) = \frac{x^2}{(1-x^2)^2}$

6 (c) Find the radii of convergence of the series in parts 4a and 4b.

① → 6 anything not remotely similar to  $\sum_{n=0}^{\infty} x^{2n}$

-2  $\sum (-1)^n x^{2n}$

-5  $\sum \frac{x^{2n}}{(2n)!}$

-4  $\sum \frac{1}{x^{2n}}$

② -8 does  $\sum a_n^2 = (\sum a_n)^2$

-3 Takes a der. but works it

-5 Takes an integral

③  $R=1$  why?

-2 Just  $R=1$  explained for ①

5. (25 pts) Determine whether the series converges. If it converges, find the sum.

Σ (a)  $\sum_{n=3}^{\infty} \frac{4}{n^2-4}$  (a) -1 Does telescopic but gets

Σ (b)  $\sum_{n=2}^{\infty} \frac{1}{\ln n}$  wrong sum

Σ (c)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$

Σ (d)  $\sum_{n=1}^{\infty} \frac{1}{n+\ln n}$

Σ (e)  $\sum_{n=1}^{\infty} \frac{(-1)^n \cos \pi n}{n}$

(b) -3 tries to use integral test  
-4 tries to use l'Hôpital

(c) -3 Takes abs val & doesn't split up series

-4 alt. series backwards

(d) -3 Compares the wrong way

(e) -2 Says fails alt. series test

-4 Attempts to use the ratio test

6. (Bonus 10 pts) A function  $f(x)$  is defined by

$$f(x) = 1 + 2x + 3x^2 + x^3 + 2x^4 + 3x^5 + x^6 + 2x^7 + 3x^8 + \dots$$

Find the interval of convergence of the series and find an explicit formula for  $f(x)$ .