

You should work through this as you would an actual exam. Time yourself, and try to solve each problem alone without using your notes or your book.

1. Find the area enclosed by the given curves.

(a)  $y = x^2 + x - 16, y = \frac{16}{x}$

(b)  $y = 2^x, y = \log_2 x, x = 2, x = 8$

(c)  $x = 4 - y^2, y = -\frac{x^2}{24} + 2$

(d)

$$\begin{cases} x = t^2 - t \\ y = t^4 - 3t^3 + 3t^2 - t \end{cases}$$

2. Find the volume of the solid formed by rotating the given region about the indicated axis:

(a) The region bounded by  $y = \log_2 x, y = (x - 1)^2$  about  $x = 3$

(b) The region bounded by  $y = -x^4 + 8x^3 - 20x^2 + 16x, y = 0$  about the  $y$ -axis.

(c) The region bounded by  $y = 4 - x^2, y = 0$  about the  $x$ -axis.

3. Prove that the volume of a sphere is  $\frac{4}{3}\pi r^3$ , where  $r$  is the radius.
4. Prove that the volume of a cone is  $\frac{1}{3}\pi r^2 h$ , where  $r$  is the base radius and  $h$  is the height.
5. Prove that the volume of a square pyramid is  $\frac{1}{3}x^2 h$ , where  $x$  is the base edge length and  $h$  is the height.
6. Find the length of the curve:

(a)

$$\begin{cases} x = \ln t - 2t^2 \\ y = 4t \end{cases}, 1 \leq t \leq 2$$

(b)

$$y = x^2, -2 \leq x \leq 1$$

(c)

$$r = \sin \theta$$

(Use an integral. You may check your work a known arc length formula from geometry.)

7. For the function  $f(x) = x + \cos x$ ,
- (a) Find the average value of  $f(x)$  on  $[0, 2\pi]$ .
  - (b) Find a number  $c$  such that  $f(c)$  equals your answer from part 7a.
  - (c) Illustrate the geometric interpretation of the answer to part 7b using areas.
8. Suppose a red carpet is being rolled up by a machine that holds the rolled portion in place while dragging the unrolled portion over rough concrete. Determine the amount of work done if the coefficient of friction between the carpet and the concrete is 0.62, and the carpet is 100' long with a weight of 250 lbs.
9. Suppose the work required to stretch a spring from a natural length of 20cm to 40cm is 15J. Determine the work required to stretch the spring from 40cm to 60cm.
10. Suppose a plate in the shape of an equilateral triangle with side lengths 6m is submerged vertically in water with the top edge parallel to the surface and 1m below the surface. Find the hydrostatic force on one side of the plate.
11. Suppose a lamina has boundaries given by  $y = e^x, y = 0, x = 0, x = 3$ . Find its centroid.