Exam 1

Name: Grading Guide

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

15 1. Use the definition of the definite integral to find

$$\int_3^6 x^3 \mathrm{d}x$$

-1 minor arith

-4 Uses $x_k = \frac{3k}{n}$

-4 Does $(a+b)^3 = a^3 + b^3$ or similar

-6 Gets the def. but doesn't go anywhere with it

-4 Applies to $f(x) = x^2$?

-3 Barks a summation formula

-3 doesn't try to evaluate the limit

-4 messes up summation in def

2. Use a midpoint sum with n subintervals to approximate

$$\int_3^6 x^3 dx$$

10

for

(a) $n = 6$

(b) $n = \cancel{30}$ 12

(c) ~~$n = 100$~~

(d) ~~$n = 999$~~

Write out the complete sum for part 2a, including the values of $a, b, \Delta x$, and all the x_k's.

-1 Typo in calc

-2 Does right-hand

-2 Typo in (b) w/ no work shown

-3 Gets the ans but writes out the Σ sum incorrectly

-3 Includes n+1 terms in the sum or only n-1

-3 Picks x_k's incorrectly

-3 no (2)

-4 Barks the sum formula completely but still writes down some x_k's

-5 Uses FTC & Riemann Sum

-4 No sum written out

-6 no approx given

−4 Uses a diff. rule −1 wrong variable −1 no +C −3 total

30

3. Evaluate the following indefinite integrals:

(a) $\int \frac{1}{4^x} dx$

(b) $\int \frac{1}{\sqrt{1-x^2}} dx$

(c) $\int \csc^2 t dt$

(d) $\int \sin\theta d\theta$

(e) $\int \frac{y^2-5y+3}{\sqrt{y}} dy$

(f) $\int \tan x dx$

(a) does $\frac{4^{-x}}{\ln 4}$, $\frac{1}{4^x \ln 4}$ or similar

(b) −3 wrong arcsine

(c) −1 sign error

(d)

(e) −2 broke rule slightly
−3 rewrites as powers but no rule

(f) −3 Does a subs but not $u = \cos x$
−1 no diff. var

10

4. Suppose water drips from a leaky drainage pipe onto the floor below at a rate given (in ml/min) by $r(t) = 5 - \frac{5}{t^2+1}$. Find

$$\int_0^{2880} r(t)\mathrm{d}t$$

Give units and interpret your answer.

-1 slightly
-2 moderately
-3 totally
max 1.5x above R.

-0 Ports an integrule
Forgets to say
"during 1st 2880 min, or similar"

-3 No interp
-2 Bad units
-2 Uses degrees
-5 No antider
-2 Fake work
-1 verb issues
-2 Interps as rate chg not total chg

10

5. Suppose the velocity of a particle at time t is given (in m/s) by $v(t) = t^2 + t - 12$. Find the distance traveled by the particle during the first 6 seconds.

-4 Finds displacement

-1 Sign error

-1 Works an app. of rule (max -2)

-7 Tries to set up as a motion problem w/ s(b) etc.

-2 Fake work

-3 Splits into two integrals but doesn't take abs. val. properly

-1 Typo

-2 Bad limits of int

-1 Doesn't show plug

6. Evaluate:

$$\int_{-3}^{4} \frac{\tan^{-1} t - 1}{t^2 + 1} dt$$

15

-10 Applies FTC w/ no subs.

-3 [illegible] Tries a subs but does the wrong one

-1 or • error in inner integ. rule

-5 Seems to use subs but doesn't try anything

-1 Uses subs w/ old limits of int or sim.

-2 Gets correct [illegible] u but doesn't subs correctly

7. Evaluate:

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x \frac{\sqrt{1-t^2}}{\sin^2(t^3)}\mathrm{d}t$$

-1 Doesn't change the variable

-3 Leaves in dx in integration sign

-5 "Wrong"

-2 Leaves in "dx"