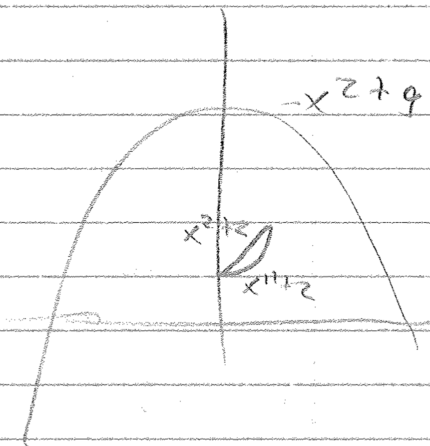
Basic Info

ⓐ $M_y = \rho \int_a^b \frac{f(x)+g(x)}{2}(f(x)-g(x))\,dx \quad M=$

$M_x = \rho \int_a^b x(f(x)-g(x))\,dx$



$m = \rho \int_a^b (f(x)-g(x))\,dx = \rho A$

$\bar{x} = M_y/m$
$\bar{y} = M_x/m$

Moments are additive

The moment wrt x-axis of the whole pate

$M_{xp} = \rho \int_{-3}^{3} \frac{-x^2+9}{2}(-x^2+9)\,dx$

$= \frac{\rho}{2}\int_{-3}^{3}(x^4-18x^2+81)\,dx = \frac{\rho}{2}\left[\frac{x^5}{5}-\frac{18x^3}{3}+81x\right]_{-3}^{3}$

$= \rho\,(243/5 - 162 + 243) = \frac{648}{5}\rho$

The moment wrt y-axis of the whole plate

$M_{yp} = \rho\int_{-3}^{3} x(-x^2+9)\,dx = \rho\int_{-3}^{3}(-x^3+9x)\,dx$

$= \rho\left[\frac{-x^4}{4}+\frac{9x^2}{2}\right]_{-3}^{3} = \rho\left(\left(\frac{-81}{4}+\frac{81}{2}\right)-\left(\frac{-81}{4}+\frac{81}{2}\right)\right)$

$= 0$

The mass of the whole plate

$m_p = \rho\int_{-3}^{3}(-x^2+9)\,dx = \rho\left[\frac{-x^3}{3}+9x\right]_{-3}^{3} = 2\rho(-9+27)$

$= 36\rho$

The moment wrt x-axis of the hole

$$M_{xh} = \frac{\rho}{2}\int_0^1 ((x^2+2)+(x^{11}+2))((x^2+2)-(x^{11}+2))\,dx$$

$$= \frac{\rho}{2}\int_0^1 (x^{11}+x^2+4)(-x^{11}+x^2)\,dx$$

$$= \frac{\rho}{2}\int_0^1 (-x^{22}+\cancel{x^{13}}-\cancel{x^{13}}+x^4-4x^{11}+4x^2)\,dx$$

$$= \frac{\rho}{2}\left[\frac{-x^{23}}{23}-\frac{4x^{12}}{12}+\frac{x^5}{5}+\frac{4x^3}{3}\right]_0^1$$

$$= \frac{\rho}{2}\left(\frac{-1}{23}-\frac{4}{12}+\frac{1}{5}+\frac{4}{3}\right) = \frac{133}{230}\rho$$

The moment wrt y-axis of the hole

$$M_{yh} = \rho\int_0^1 x((x^2+2)-(x^{11}+2))\,dx = \rho\int_0^1(-x^{12}+x^3)\,dx$$

$$= \rho\left[\frac{-x^{13}}{13}+\frac{x^4}{4}\right]_0^1 = \rho\left(\frac{-1}{13}+\frac{1}{4}\right) = \frac{9}{52}\rho$$

The mass of the hole

$$m_h = \rho\int_0^1((x^2+2)-(x^{11}+2))\,dx = \rho\int_0^1(-x^{11}+x^2)\,dx$$

$$= \rho\left[\frac{-x^{12}}{12}+\frac{x^3}{3}\right]_0^1 = \rho\left(\frac{-1}{12}+\frac{1}{3}\right) = \frac{1}{4}\rho$$

The moments, mass, and centroid of the lamina with the hole knocked out.

$$M_{x\ell} = M_{xp} - M_{xh} = \frac{648}{5}\rho - \frac{133}{230}\rho = \frac{5935}{46}\rho$$

$$M_{y\ell} = M_{yp} - M_{yh} = 0 - \frac{9}{52}\rho = \frac{-9}{52}\rho$$

$$m_\ell = m_p - m_h = 36\rho - 1/4\,\rho = 143/4\,\rho$$

$$\overline{x_\ell} = M_{y\ell}/m_\ell = -9/52\,\rho \,/\, 143/4\,\rho = \frac{-9}{1859} \cong -0.00484$$

$$\overline{y_\ell} = M_{x\ell}/m_\ell = \frac{5935}{46}\rho \,/\, 143/4\,\rho = \frac{2374}{989} \cong 3.609$$

(b) The centroid will remain unchanged $\Leftrightarrow$ the centroid of the hole lies directly on top of the centroid of the plate.

pf: If $\overline{x_p} = \overline{x_h}$ then $\frac{M_{yp}}{m_p} = \frac{M_{yh}}{m_h}$.

So $\overline{x_\ell} = M_{y\ell}/m_\ell = \frac{M_{yp} - M_{yh}}{m_p - m_h}$

and $M_{yp} = m_p M_{yh}/m_h \Rightarrow$

$$\overline{x_\ell} = \frac{m_p M_{yh}/m_h - M_{yh}}{m_p - m_h} = \frac{M_{yh}(m_p/m_h - 1)}{m_p - m_h}$$

$$= \frac{M_{yh}}{m_h}\frac{(m_p - m_h)}{(m_p - m_h)} = \frac{M_{yh}}{m_h} = \overline{x_h} = \overline{x_p}$$

By a similar argument if $\overline{y_p} = \overline{y_h}$, then $\overline{y_\ell} = \overline{y_h} = \overline{y_p}$.

Conversely, if $\overline{x_\ell} = \overline{x_h}$, then
$\overline{x_p} = \frac{M_{yp}}{m_p} = \frac{M_{y\ell} + M_{yh}}{m_\ell + m_h}$. Also $\overline{x_\ell} = \overline{x_h} \Rightarrow$

$\frac{M_{y\ell}}{m_\ell} = \frac{M_{yh}}{m_h} \Rightarrow M_{y\ell} = \frac{m_\ell M_{yh}}{m_h} \Rightarrow \overline{x_p} =$

$$\frac{m_\ell M_{yh}/m_h + M_{yh}}{m_\ell + m_h} = \frac{M_{yh}(m_\ell/m_h + 1)}{m_\ell + m_h} = \frac{M_{yh}(m_\ell + m_h)}{m_h(m_\ell + m_h)}$$

$$= \frac{M_{yh}}{m_h} = \overline{x_h} = \overline{x_\ell}.$$

By a similar argument if $\overline{y_\ell} = \overline{y_h}$, then $\overline{y_p} = \overline{y_\ell} = \overline{y_h}$.