1. Use the definition of the limit to show that

$$\lim_{x \to 0} x^3 = 0$$

2. Find

$$\lim_{x \to 5} \frac{x^{20} - 5^{20}}{x^{10} - 5^{10}}$$

3. Find

$$\lim_{x \to \infty} \left(\cos \left(\frac{\pi}{x} \right) \right)^{(x^2)}$$

4. Find

$$\lim_{x \to -\infty} \frac{x^9 + x^6 + x^3 + 1}{3x^9 + 2x^6 + x^3 + 1}$$

5. Show that

$$\lim_{x \to \infty} \frac{\sin x}{[\![x]\!]} = 0$$

6. Show that

$$\lim_{x \to -2} \frac{|x + 2|}{x + 2}$$

   DNE.

7. Draw the graph of a function with an infinite discontinuity at $x = 3$, a removable discontinuity at the origin, and a jump or break discontinuity at $x = -3$.

8. The charge in a battery after t hours of use is given (in Coulombs) by

$$C(t) = 25000 - 3600t$$

   Use the definition of the derivative to find $C'(4)$. Give units and interpret your answer.

9. Use the definition of the derivative to find

$$\frac{\mathrm{d}}{\mathrm{d}x} \sqrt{x}$$

10. Use the definition of the derivative to find the formula for the tangent to the graph of $f(x) = \frac{1}{x^2}$ at $x = 2$.

11. Draw a possible graph of $f'(x)$, assuming the following:

    - $f(x)$ is increasing on $[-5, 5]$ and decreasing everywhere else
    - $f(x)$ is concave up when x is negative and concave down when x is positive

12. Sketch a possible graph of $f(x)$, assuming the following:

- $f'(x)$ is positive when $-4 < x < 0$ and when $x > 4$
- $f'(x)$ is negative when $0 < x < 4$ and when $x < -4$
- $f'(x)$ is decreasing when $-2 < x < 2$ and increasing elsewhere

13. State and prove the difference rule for derivatives.

14. Find
$$\frac{d}{dx} \frac{2^x}{\sqrt{x}}$$

15. Suppose the population of California condors t years after the start of a program to reintroduce them into the wild is given by
$$C(t) = \left(10e^{0.01t} + 5t\right)\left(\sin(\pi t) + 2\right)$$
Find the rate at which the condor population is changing in 1995 if the program started in 1980.

16. Find
$$\frac{d}{dx} \ln\left|x^4 + e^x\right|$$

17. Find the tangent to $r = f(\theta)$ when $(r, \theta) = \left(\frac{\pi^2}{4}, \frac{\pi}{2}\right)$, where
$$f(\theta) = \theta^2$$

18. Find the tangent to $x = \sin t$, $y = \cos t$ when $t = \pi$.

19. Find y' if
$$x^2 \sin y^2 = y^2 \sin^2 x$$

20. Find
$$\frac{d}{dx} (\ln x)^{(\ln x)}$$

21. Approximate $\sqrt{8}$ using local linear approximation. Perform all arithmetic by hand, and use a graph to illustrate whether your answer is an overestimate or an underestimate.

22. Approximate $\sqrt{8}$ using Newton's method. Show all your work for the first two steps, then list all successive iterations until they converge to within the maximum accuracy supported by your calculator.

23. Graph $f(x) = x^3 - x$ by hand. Show the calculation of all maxima, minima, and inflection points.

24. A cat burglar tosses a grappling hook over the outer wall of a gated community and pulls the line taut. The burglar pulls the line at a rate of 2.5 ft/s, and the wall is 12 feet tall. Determine how fast the hook is moving when it is 5 feet from the wall, assuming it moves horizontally along the ground as the line is pulled in.

25. Suppose an ice cream cone is to be made using 20 in² of waffle wafer. Find the dimensions of the cone of maximum capacity. (Use the fact that a cone with radius r, height h, and slant height H has surface area $\pi r H$ and volume $\frac{1}{3}\pi r^2 h$).