Practice Exam 4

1. $0 = x^4 - 24x^2 + 144$

$0 = (x^2 - 12)^2$

$x = \pm\sqrt{12}$ zeros

$f'(x) = 4x^3 - 48x$

$= 4x(x^2 - 12)$

$x = 0, \pm\sqrt{12}$ cpts

$f''(x) = 12x^2 - 48$

$0 = 12x^2 - 48$

$= 12(x^2 - 4)$

$x = \pm 2$ possible ips

$(-\infty, -\sqrt{12}) \downarrow$

$(-\sqrt{12}, 0) \uparrow$

$(0, \sqrt{12}) \downarrow$

$(\sqrt{12}, \infty) \uparrow$

$f'(-4) = -16(4) < 0$

$f'(-1) = 44 > 0$

$f'(1) = -44 < 0$

$f'(4) = 64 > 0$

$(-\infty, -2)$ $f''(-3) = 12(5) > 0$ cup

$(-2, 2)$ $f''(0) = -4(12) < 0$ cnvn $x = \pm 2$ ips

$(2, \infty)$ $f''(3) = 12(8) > 0$ cup

$(\pm 2, 64)$ ips

$(\pm\sqrt{12}, 0)$ lmin

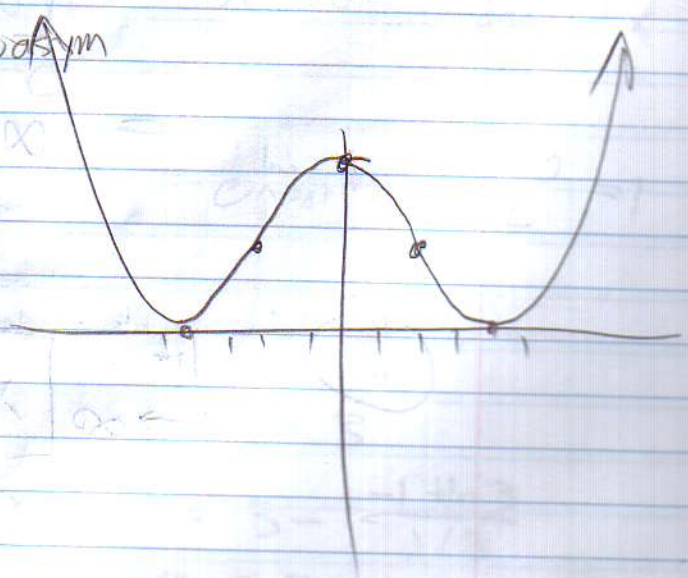
$(0, 144)$ lmax

dom $f = \mathbb{R}$ $\Rightarrow$ no vasym

f cnts

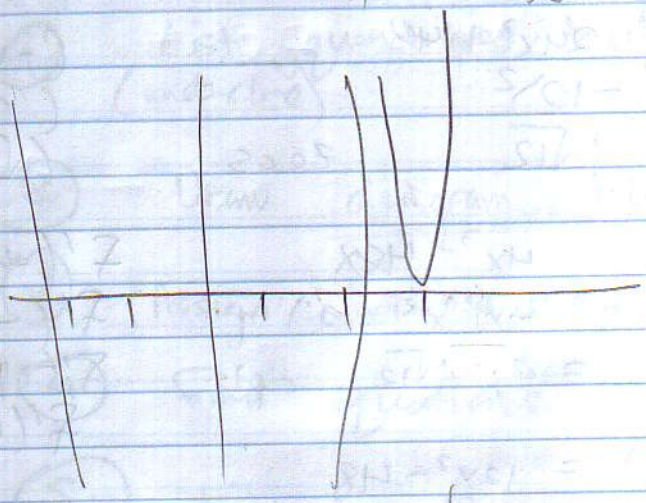
$\lim_{x \to \pm\infty} f(x) = \infty$

$\Rightarrow$ no hasym



2. $f'(x) = 30x^4 - 180x^3 + 120x^2 + 900x + 130$

$f''(x) = 120x^3 - 540x^2 + 240x + 900$



deg 3

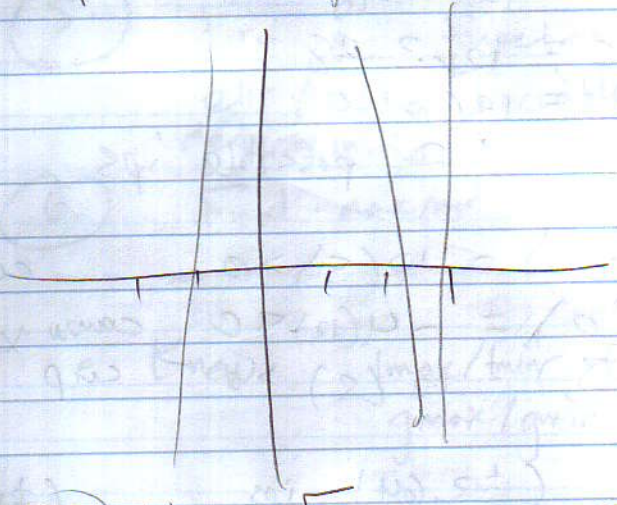
opts

$x = -2.23, 2.23, 3$

only zeros of f'

lmax $x = -2.23$

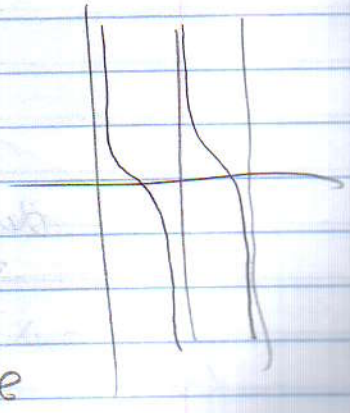
lmin $x = 2.23$



ips:

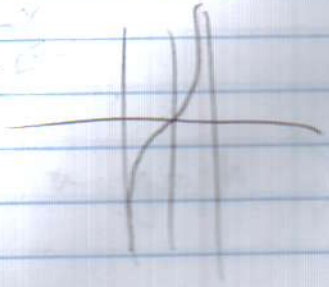
$x = -1, 2.4, 3$

3. (a) $\lim_{x \to \infty} \left[x^3 \cot\left(\frac{\pi}{x}\right) \right]$



$= \infty \cdot \infty = \infty$ not indeterminate

(b) $\lim_{x \to \infty} \left[x^3 \cdot \tan\left(\frac{\pi}{x}\right) \right]$



$= \infty \cdot 0$

$$\Rightarrow \lim_{x\to\infty} \frac{x^3}{\cot(\pi/x)} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{3x^2}{-\frac{\pi}{x^2}\left(-\csc^2\left(\frac{\pi}{x}\right)\right)}$$

$$= \lim_{x\to\infty} \frac{-3x^4 \sin^2\left(\frac{\pi}{x}\right)}{\pi}$$

$$= \lim_{x\to\infty} \frac{\tan(\pi/x)}{1/x^3} \overset{0/0}{=} \lim_{x\to\infty} \frac{\sec^2(\pi/x)\left(-\pi/x^2\right)}{-3/x^4}$$

$$= \lim_{x\to\infty} \frac{\pi \sec^2(\pi/x)}{3/x^2} = \lim_{x\to\infty} \frac{\pi x^2}{\cos^2(\pi/x)} = \frac{\pi\infty}{1}$$

$$= \infty$$

4. $y = \sin x^{\sin x}$

$$\ln y = \sin x \ln \sin x$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \sin x \ln \sin x$$

$$= \lim_{x\to 0^+} \frac{\ln \sin x}{1/\sin x} \overset{-\infty/\infty}{=} \lim_{x\to 0^+} \frac{\cos x/\sin x}{-\csc x \cot x}$$

$$= \lim_{x\to 0^+} -\sin x = 0$$

$$\lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y} = e^{\lim_{x\to 0^+} \ln y} = e^0 = 1$$

5 (done in class notes, not presented)

6. $f(x) = -1 + \ln x \qquad x_0 = 3$

$f'(x) = 1/x$

$$x_1 = 3 - \frac{-1+\ln 3}{1/3}$$

$$\approx 2.704163134$$

$$x_2 = 2.704163134 - \frac{-1 + \ln 2.704163134}{1/2.704163134}$$

$$= 2.718245009$$

$$x_3 = 2.718281828$$

$$x_4 = 2.718281828$$