Exam 4

Name: KEY

Each problem is worth 25 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

1. Graph by hand:

$$f(x) = \frac{x^2-4}{x^2-9}$$

Show all intercepts, asymptotes, intervals of increase/decrease, local extrema, intervals of concavity, and inflection points.

$0 = f \quad x = \pm 2$ x incpt

$f(0) = 4/9$ y incpt

$x = \pm 3$ vasym

$\lim_{x\to -3^-} f = 5/0^+ = \infty$

$\lim_{x\to -3^+} f = 5/0^- = -\infty$

$\lim_{x\to 3^-} f = 5/0^- = -\infty$

$\lim_{x\to 3^+} f = 5/0^+ = \infty$

$\lim_{x\to\pm\infty} \frac{x^2-4}{x^2-9} = \lim_{x\to\pm\infty} \frac{1-4/x^2}{1-9/x^2} = 1$

hasym $y=1$

$f' = \frac{2x(x^2-9)-(x^2-4)2x}{(x^2-9)^2}$

$= \frac{2x^3-18x-2x^3+8x}{(x^2-9)^2}$

$= \frac{-10x}{(x^2-9)^2}$

$0 = f' \Rightarrow x = 0$

denom always +, ignore

$(-\infty,-3)$ $f' > 0$, ↗

$(-3,0)$ $f' > 0$, ↗

$(0,3)$ $f' < 0$, ↘ $f(0) = 4/9$ lmax

$(3,\infty)$ $f' < 0$, ↘

$f'' = \frac{-10(x^2-9)^2+10x(2)(x^2-9)(2x)}{(x^2-9)^4}$

$= \frac{-10(x^2-9)+40x^2}{(x^2-9)^3}$

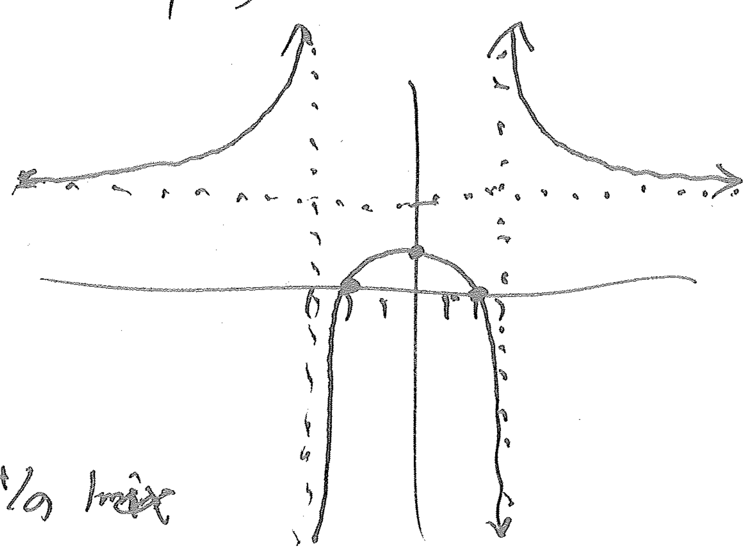
$= \frac{30x^2+90}{(x^2-9)^3} = \frac{30(x^2+3)}{(x^2-9)^3}$

$x = \pm 3$ poss. ips

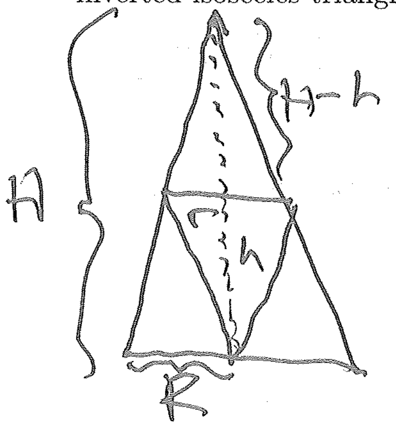
$(-\infty,-3)$ f'' + cup

$(-3,3)$ f'' − cDN

$(3,\infty)$ f'' + cup



3. Find the largest proportion of an isosceles triangle that can be occupied by an inverted isosceles triangle contained inside of it.



$$\frac{r}{R} = \frac{H-h}{H} \qquad r = R\left(\frac{H-h}{H}\right)$$

$$A_B = RH$$

$$A_S = r\cdot h$$

$$= R\left(\frac{H-h}{H}\right)h$$

$$= \frac{RHh}{H} - \frac{Rh^2}{H}$$

$$= Rh - \frac{Rh^2}{H}$$

$$\text{dom } A_S = [0, H]$$

$$0 = A_S' = R - \frac{2Rh}{H}$$

$$R = \frac{2Rh}{H} \qquad h = \frac{RH}{2R} = H/2 \in \text{dom } A_S$$

$$A(0) = 0$$

$$A(H/2) = RH/2 - \frac{R(H/2)^2}{H} = RH/2 - RH/4 = RH/4$$

max

$$A(H) = 0$$

$$\frac{A_S}{A_B} = \frac{\frac{RH}{4}}{RH} = \frac{1}{4}$$