Name KEY #2

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

1. Find the derivative:

   (a) $\frac{\mathrm{d}}{\mathrm{d}x} \ln \tan x$

   (b) $\frac{\mathrm{d}}{\mathrm{d}x} \sin \sqrt{x}$

   (c) $\frac{\mathrm{d}}{\mathrm{d}x} e^x \cot x$

   (d) $\frac{\mathrm{d}}{\mathrm{d}x} (\csc x)^x$

(a) [illegible] $\frac{\sec^2 x}{\tan x}$

(b) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$

(c) $e^x \cot x - e^x \csc^2 x$

(d) $\ln y = x \ln \csc x$

$\frac{y'}{y} = \ln \csc x + x\left(\frac{-\csc x \cot x}{\csc x}\right)$

$y' = (\ln \csc x - x \cot x)(\csc x)^x$

3. Find the slope of the tangent line to the graph given (in polar coordinates) by $r = \theta^2$ at $\theta = \frac{\pi}{2}$.

$$y = \theta^2 \sin\theta$$

$$x = \theta^2 \cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\theta\sin\theta + \theta^2\cos\theta}{2\theta\cos\theta - \theta^2\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{\pi\sin\frac{\pi}{2} + \frac{\pi^2}{4}\cos\frac{\pi}{2}}{\pi\cos\frac{\pi}{2} - \frac{\pi^2}{4}\sin\frac{\pi}{2}}$$

$$= \frac{\pi}{-\pi^2/4} = \frac{-4}{\pi}$$

5. Use local linear approximation to estimate $\ln 3$. Use a graph to show whether your approximation is an overestimate or an underestimate.

$f(x) = \ln x$

$a = e$

$x = 3$

$f(x) \approx f(a) + f'(a)(x-a)$

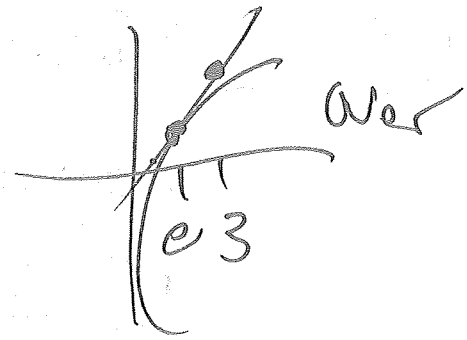
$= \ln e + \frac{1}{e}(x-e)$

$= 1 + \frac{x}{e} - 1$

$= \frac{x}{e}$

$f(3) = 3/e \approx 3/2.718281828 \approx$

1.1036



7. Find the global minimum and global maximum of $f(x) = x^3 - 12x$ on $[0, 4]$.

$0 = f'(x) = 3x^2 - 12$

$= 3(x^2 - 4) = 3(x-2)(x+2)$

$x = \pm 2$

$f(0) = 0$

$f(2) = -16$ gmin

$f(4) = 16$ gmax