- Speed up / slow down
- Know what marginal means

3.4) Derivatives of trig function

- $\frac{d}{dx} \sin x = \cos x$ (apply)
- $\frac{d}{dx} \cos x = -\sin x$ (apply)
- Other trig diff. rules (apply, prove)

p 177-178 # 26, 28, 32, 36, 38, 42

p 255-257 # 6, 8, 36, 40, 44, 48 ab, 60, 64, 68, 76

4:53:30 Practice Exam 2

1. $S'(2) = \lim_{x \to 2} \frac{S(x) - S(2)}{x - 2}$

x	$\frac{S(x) - S(2)}{x - 2}$
0	-0.3795
1	-0.599
3	-0.631
4	-0.6405

(1 and 3) avg -0.615

On the 1st day of March, the sun is rising earlier at a rate of 0.615 hrs/month = 36.9 min/month

2. $$\lim_{x \to \pi/2} \frac{e^{\cos x} - e^{\cos(\pi/2)}}{x - \pi/2}$$

x	$\dfrac{e^{\cos x} - 1}{x - \pi/2}$
$\pi/2 - 0.1$	-1.0869
$\pi/2 - 0.01$	-1.0869
$\pi/2 - 0.001$	-1.08689
$\pi/2 + 0.001$	"
$\pi/2 + 0.01$	"
$\pi/2 + 0.1$	"

$$m \cong -1.0869$$

$$y - 1 = -1.0869(x - \pi/2)$$
$$y = -1.0869x + \pi/2 + 1$$

3. $$f'(x) = \lim_{h \to 0} \frac{\sqrt{5(x+h)+1} + (x+h)^2 - \sqrt{5x+1} - x^2}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} + \lim_{h \to 0} \frac{\sqrt{5x+5h+1} - \sqrt{5x+1}}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} + \lim_{h \to 0} \frac{5x + 5h + 1 - 5x - 1}{h(\sqrt{5x+5h+1} + \sqrt{5x+1})}$$

$$= \lim_{h \to 0} (2x + h) + \lim_{h \to 0} \frac{5}{\sqrt{5x+5h+1} + \sqrt{5x+1}}$$

$$= 2x + \frac{5}{2\sqrt{5x+1}}$$

4. $(0, \infty)$: $f'(x) = \frac{d}{dx} x = 1$

$(-\infty, 0)$: $f'(x) = \frac{d}{dx} -x = -1$

0 :

$$f'(0) = \lim_{h \to 0} \frac{|0+h| - |0|}{h}$$

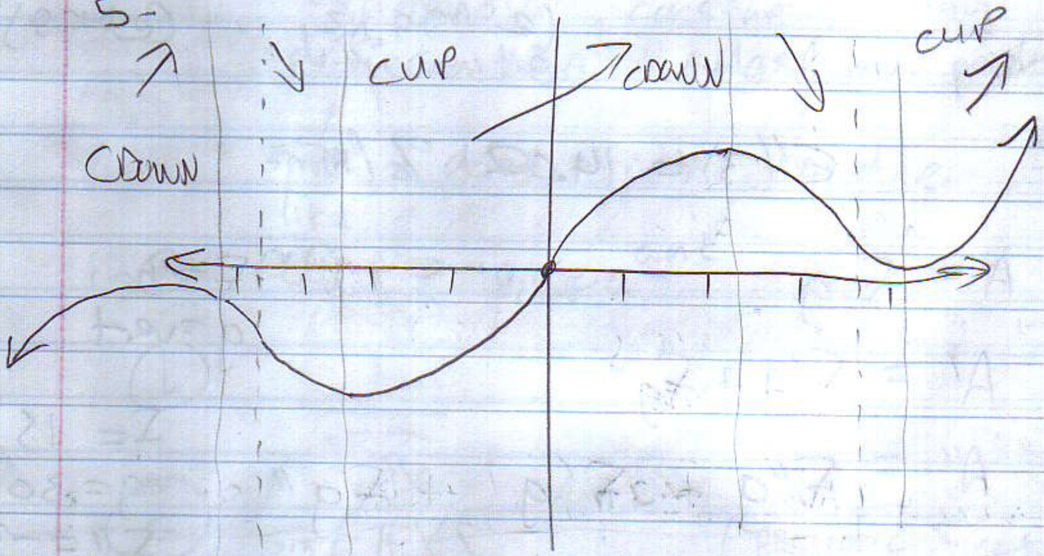
$$= \lim_{h \to 0} \frac{|h|}{h}$$

$$\lim_{h \to 0^+} \frac{|h|}{h} = \lim_{h \to 0^+} \frac{h}{h} = \lim_{h \to 0^+} 1 = 1$$

$$\lim_{h \to 0^-} \frac{|h|}{h} = \lim_{h \to 0^-} \frac{-h}{h} = \lim_{h \to 0^-} -1 = -1$$

So $\lim_{h \to 0} \frac{|h|}{h}$ DNE $\Rightarrow$ f not diff.able at 0.

5.



6. $= \frac{d}{dx} \frac{x^{1/2} x^{1/3}}{x^4} = \frac{d}{dx} x^{5/6 - 24/6}$

$= \frac{d}{dx} x^{-19/6} = -\frac{19}{6} x^{-25/6}$

$= \frac{-19}{6\sqrt[6]{x^{25}}}$

10:03, 11:04

7.

$$E'(t) = \frac{d}{dt} \frac{100e^t}{e^t+99} = \frac{f}{g}$$

$f' = 100e^t$

$g' = e^t$

$$\frac{f'g - fg'}{g^2} = \frac{100e^t(e^t+99) - 100e^t e^t}{(e^t+99)^2}$$

$$= \frac{100e^t(99)}{(e^t+99)^2} = \frac{9900e^t}{(e^t+99)^2}$$

$E'(3) = 14.02$ %/min

8 $A = fg$ 　 f = hori, g = vert

$A' = f'g + fg'$ 　 $f = 15$

$A'' = f''g + 2f'g' + fg''$ 　 $g = 30$

$A'' = -30(30) + 15(55)$ 　 $f'' = -30$, $g'' = 55$

$= -75$ m²/s²

Also $A' = -75 m^2/s$, so shrinkage is speeding up

9

$$\frac{d}{dx} \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\frac{d}{dx} \frac{1}{\cos^2 x} = \frac{0(\cos^2 x) - 1(\cos x(-\sin x) + \cos x(\sin x))}{\cos^4 x}$$

$$= \frac{2\cos x \sin x}{\cos^4 x} = \frac{2\sin x}{\cos^3 x}$$

11:11

10 $ex^{e-1} + e^x + e + e + 1$

$= ex^{e-1} + e^x + 2e + 1$

$eq^{e-1} + e^q + e + e + 1$

$= eq^{e-1} + e^q + 2e + 1$

The Chain Rule

3.5

We combine basic rules (power, trig, exponential, constant) using arithmetic differentiation rules (sum, product, quotient)

This does not handle functions like

(a) $p(t) = 10{,}000e^{0.01t}$ (bank acct w/ 1% interest)

(b) $\frac{1}{\sqrt[3]{5x}}$

(c) $\sin\left(\frac{\pi x}{12}\right)$ (periodic function with period = 24)

These can all be viewed as compositions of functions for which we have basic rules

(a) $f(t) = 10{,}000e^t$ $g(t) = 0.01t$ $f(g(t)) = 10{,}000e^{0.01t}$

(b) $f(x) = \frac{1}{\sqrt[3]{x}}$ $g(x) = 5x$

$f(g(x)) = \frac{1}{\sqrt[3]{5x}}$

(c) $f(x) = \sin x$

$g(x) = \frac{\pi x}{12}$ $f(g(x)) = \sin\left(\frac{\pi x}{12}\right)$