Name: KEY #2

Each problem is worth 15 points; work at least seven problems. You may work an eighth problem for extra credit. If you work more than eight, I will only grade the first eight. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

1. Find the derivative:

   (a) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{x}{e^x+x^e}$

   (b) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{\sqrt[4]{x^3}}{\sqrt[3]{x^4}}$

   (c) $\frac{\mathrm{d}}{\mathrm{d}x}e^x \tan x$

(a) $$\frac{1(e^x+x^e) - x(e^x + ex^{e-1})}{(e^x+x^e)^2}$$

(b) $$= \frac{d}{dx} x^{3/4-4/3} = \frac{d}{dx} x^{9/12-16/12}$$

$$= \frac{d}{dx} x^{-7/12} = -\frac{7}{12} x^{-19/12}$$

(c) $$e^x \tan x + e^x \sec^2 x$$

3. Use the definition of the derivative to find

$$\frac{d}{dx}\frac{1}{x^2}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2}$$

$$= \lim_{h\to 0} \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2} = \lim_{h\to 0} \frac{-2xh - h^2}{hx^2(x+h)^2}$$

$$= \lim_{h\to 0} \frac{-2x - h}{x^2(x+h)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

5. (a) Use the definition of the derivative to prove the reciprocal rule $\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{g(x)} = -\frac{g'(x)}{g(x)^2}$

(b) Use the reciprocal rule and the product rule to prove the quotient rule.

(a) $$\lim_{h\to 0} \frac{\frac{1}{g(x+h)} - \frac{1}{g(x)}}{h} = \lim_{h\to 0} \frac{g(x)-g(x+h)}{hg(x)g(x+h)}$$

$$= \lim_{h\to 0} - \frac{g(x+h)-g(x)}{h} \lim_{h\to 0} \frac{1}{g(x)g(x+h)}$$

$$= -g'(x) \frac{1}{g(x)^2} = -\frac{g'(x)}{g(x)^2}$$

(b) $$\frac{d}{dx}\frac{f}{g} = \frac{d}{dx} f\left(\frac{1}{g}\right) = \left(\frac{d}{dx}f\right)\frac{1}{g} + f\left(\frac{d}{dx}\frac{1}{g}\right)$$

$$= \frac{f'}{g} + f\left(-\frac{g'}{g^2}\right) = \frac{f'g - fg'}{g^2}.$$

7. Suppose the position of a particle after t seconds is given by $f(t) = t^3 - 18t^2 + 96t$. Determine when the particle is speeding up and when it is slowing down.

$$0 = f'(t) = 3t^2 - 36t + 96$$
$$= 3(t^2 - 12t + 32)$$
$$= 3(t-8)(t-4)$$
$$t = 4, 8$$

$$0 = f''(t) = 6t - 36$$
$$t = 6$$

	$(0,4)$	$(4,6)$	$(6,8)$	$(8,\infty)$
f'	$+$	$-$	$-$	$+$
f''	$-$	$-$	$+$	$+$
sped/slow	slow	sped	slow	sped