

Name: _____

Each problem is worth 15 points; work at least seven problems. You may work an eighth problem for extra credit. If you work more than eight, I will only grade the first eight. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

1. Find the derivative:

(a) $\frac{d}{dx} \frac{x}{e^x + x^e}$

(b) $\frac{d}{dx} \frac{\sqrt[4]{x^3}}{\sqrt[3]{x^4}}$

(c) $\frac{d}{dx} e^x \tan x$

2. Differentiate:

$$\frac{d}{dx} \frac{x}{1 + \frac{x}{1+x}}$$

3. Use the definition of the derivative to find

$$\frac{d}{dx} \frac{1}{x^2}$$

4. State the product rule, and prove the product rule using the definition of the derivative.

5. (a) Use the definition of the derivative to prove the reciprocal rule $\frac{d}{dx} \frac{1}{g(x)} = -\frac{g'(x)}{g(x)^2}$
- (b) Use the reciprocal rule and the product rule to prove the quotient rule.

6. Find the equation for the tangent line to $y = 3x^2 - 6$ at $x = 2$.

7. Suppose the position of a particle after t seconds is given by $f(t) = t^3 - 18t^2 + 96t$. Determine when the particle is speeding up and when it is slowing down.

8. For the graph of $f(x)$ given below, sketch a graph of $f'(x)$.

9. Suppose the time required for a rock climber to scale a portion of a cliff face is given (in seconds) by $T(h) = 15 + 10h + 5\sqrt{h^3}$, where h is the height scaled (in meters). Find $T(100)$ and $T'(100)$. Give units and interpret your answer.