Practice Exam 1

You should work through this as you would an actual exam. Time yourself, and try to solve each problem alone without using your notes or your book.

1. Sketch the graph of each of the parametric equations given below, and explain how they differ.

   (a) $x = \sin(2t), y = 2\sin t \cos t$

   (b) $x = \tan t, y = \tan t, -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$

2. Suppose the volume V of a balloon (in ft^3) attached to a Helium tank t seconds after opening the valve is given by $V(t) = \begin{cases} e^{2t/3} - 1 & 0 \leq t \leq 10 \\ 0 & t > 10 \end{cases}$.

   (a) Find the average growth rate of the balloon during the following time intervals:

      (i) The first second

      (ii) The first half-second

      (iii) The first tenth of a second

      (iv) The first millisecond

   (b) Use the values you calculated in 2a to estimate how quickly the the balloon is growing at the moment the valve is opened.

3. Use a table of values to estimate

$$\lim_{x \to 1} \left(\frac{x^2 - 1}{x - 1} \right)$$

4. Graph two functions $f(x), g(x)$ satisfying all of the following properties simultaneously:

   (i) $\lim_{x \to 0^+} f(x) = 5$

   (ii) $\lim_{x \to 0} f(x)$ DNE

   (iii) $\lim_{x \to 0^-} g(x) = -3$

   (iv) $\lim_{x \to 0} (f + g)(x) = 0$

5. Prove that $\lim_{x \to -3} \left(\frac{x+3}{|x+3|} \right)$ DNE.

6. State the formal definition of the limit, and use it to prove that

$$\lim_{x \to 3} (5x + 2) = 17$$

7. Find the exact value of

$$\lim_{x \to 0} \left(\frac{\sqrt{x+1} - 1}{x} \right)$$

8. Prove that $x^3 + 2x^2 + 3x + 4$ has a root in the interval $[-2, -1]$.

9. Find the exact value of

$$\lim_{x \to 5} \left(\frac{x^3 + 15x^2 + 75x + 125}{x + 5} \right)$$

10. Find the exact value of

$$\lim_{x \to -\infty} \left(\frac{3x^5 - 5x^3}{4x^5 + 3x^4} \right)$$