Name: KEY #2

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

1. For the curves with parametric equations:

$$(i)\ \begin{cases} x = \cos t \\ y = \sin^2 t \end{cases} \quad 0 \le t \le 2\pi$$

$$(ii)\ \begin{cases} x = \sec t \\ y = -\tan^2 t \end{cases} \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

   (a) Graph the curve. Show the direction in which the curve is traced out, and label any initial points and terminal points.

   (b) Find a Cartesian equation for the curve.

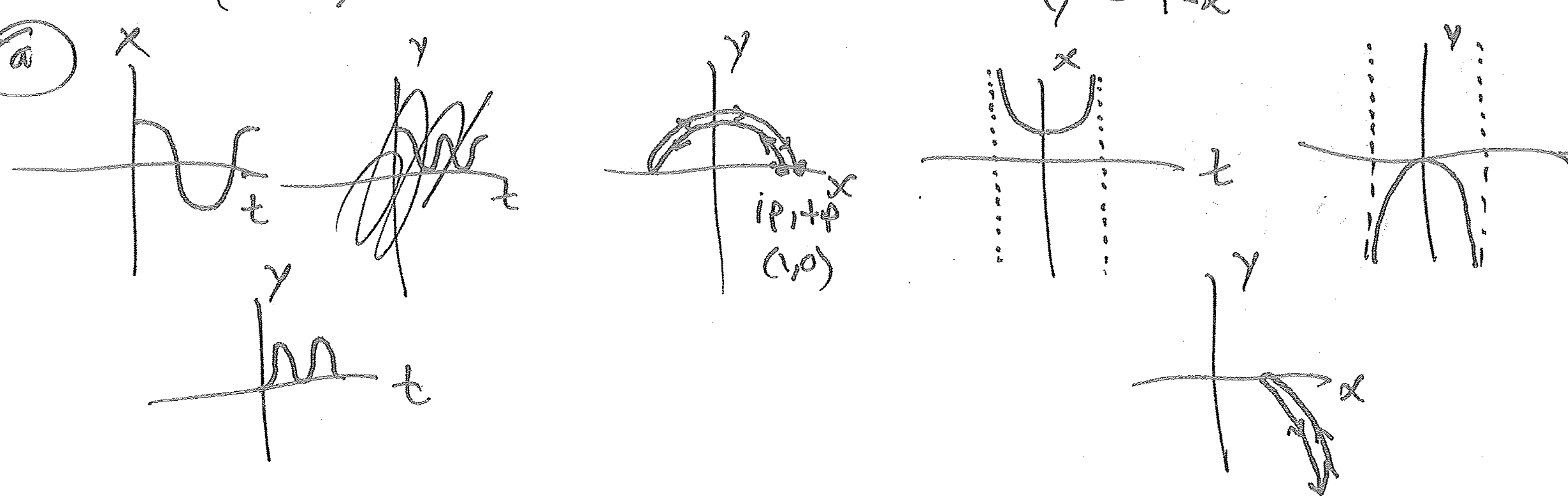
   (c) Explain the difference between the two curves.

(b) $\sin^2 t + \cos^2 t = 1$ $\qquad\qquad$ $\tan^2 t + 1 = \sec^2 t$

$y + x^2 = 1$ $\qquad\qquad$ $-y + 1 = x^2$

$y = 1 - x^2$ $\qquad\qquad$ $y = 1 - x^2$

(a)



(c) (i) is traced out twice in QI, QII, 1st R to L then L to R

(ii) is traced out twice in QIV, 1st R to L then L to R

3. Give a formula for a function $f(x)$ satisfying the following properties:

   (a) $\lim_{x \to 0} f(x)$ DNE.

   (b) $\lim_{x \to 0} 2f(x)$ DNE.

   (c) $\lim_{x \to 0} f(x)^2 = 1$.

   (d) $\lim_{x \to 0} |f(x)| = 1$.

$|x| / x$

5. State the $\epsilon - \delta$ definition of the limit, and use the definition to prove that

$$\lim_{x\to 2} 7 - 3x = 1$$

Draw a graph and label what the variables ϵ, δ, L and a represent for the above limit.

$\lim_{x\to a} f(x) = L \iff \forall \varepsilon > 0 \; \exists \delta > 0 \text{ + } 0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

Let

$\varepsilon > 0$, set $\delta = \varepsilon/3$. Then

$0 < |x-2| < \delta \Rightarrow$

$-\varepsilon/3 < x - 2 < \varepsilon/3 \Rightarrow$

$-\varepsilon/3 + 2 < x < \varepsilon/3 + 2 \Rightarrow$

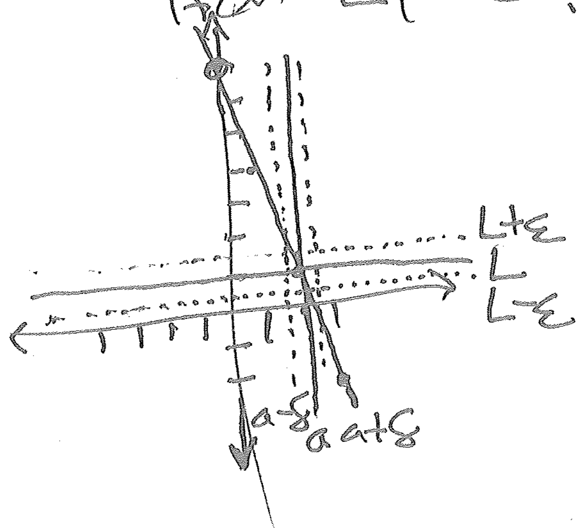
$\varepsilon - 6 > -3x > -\varepsilon - 6 \Rightarrow$

$\varepsilon + 1 > 7 - 3x > -\varepsilon + 1 \Rightarrow$

$\varepsilon > (7 - 3x) - 1 > -\varepsilon \Rightarrow$

$|(7-3x) - 1| < \varepsilon \Rightarrow$

$|f(x) - L| < \varepsilon$.



7. Approximate $\sqrt[3]{3}$ to using 5 iterations of bisection.

a	$\frac{a+b}{2}$	b	$f(a)$	$f\left(\frac{a+b}{2}\right)$	$f(b)$
1	1.5	2	−2	0.4	5
1	1.25	1.5	−2	−1	0.4
1.25	1.375	1.5	−1	−0.4	0.4
1.375	1.4375	1.5	−0.4	−0.03	0.4
1.4375	1.46875	1.5	−0.03	0.17	0.4
1.4375	1.453125	1.46875	−0.03	0.06	0.17

$1.4375 < \sqrt[3]{3} < 1.453125$