

Name: _____

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

1. For the curves with parametric equations:

$$\begin{cases} x = \cos t \\ y = \sin^2 t \end{cases} \quad 0 \leq t \leq 2\pi$$

$$\begin{cases} x = \sec t \\ y = -\tan^2 t \end{cases} \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

- Graph the curve. Show the direction in which the curve is traced out, and label any initial points and terminal points.
- Find a Cartesian equation for the curve.
- Explain the difference between the two curves.

2. Find the value of the limit:

(a)

$$\lim_{x \rightarrow 4} \frac{\frac{1}{2} - \frac{1}{x-2}}{x-4}$$

(b)

$$\lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h}$$

(c)

$$\lim_{a \rightarrow -3} \frac{a^3 + 3a^2 + a + 3}{a + 3}$$

3. Give a formula for a function $f(x)$ satisfying the following properties:

(a) $\lim_{x \rightarrow 0} f(x)$ DNE.

(b) $\lim_{x \rightarrow 0} 2f(x)$ DNE.

(c) $\lim_{x \rightarrow 0} f(x)^2 = 1$.

(d) $\lim_{x \rightarrow 0} |f(x)| = 1$.

4. Determine the intervals on which the function pictured below is continuous.

5. State the $\epsilon - \delta$ definition of the limit, and use the definition to prove that

$$\lim_{x \rightarrow 2} 7 - 3x = 1$$

Draw a graph and label what the variables ϵ , δ , L and a represent for the above limit.

6. Use limits to show that $f(x) = \frac{\cos x}{e^x}$ has a horizontal asymptote at $y = 0$.

7. Approximate $\sqrt[3]{3}$ to using 5 iterations of bisection.