Name: KGY

Each problem is worth 15 or 20 points. Work at least four of the first five problems and both of the last two problems; you may work all seven problems for extra credit. Numerical estimates are unacceptable; for full credit you must show all your work and use the indicated methods.

1. (15 pts) Find the slope the tangent line to $r = \theta^2$ when $\theta = \frac{\pi}{4}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)}$$

$$= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{2\theta\sin\theta + \theta^2\cos\theta}{2\theta\cos\theta - \theta^2\sin\theta}$$

$$= \frac{\frac{\pi}{2}\frac{\sqrt{2}}{2} + \frac{\pi^2}{16}\frac{\sqrt{2}}{2}}{\frac{\pi}{2}\frac{\sqrt{2}}{2} - \frac{\pi^2}{16}\frac{\sqrt{2}}{2}} = \frac{1+\pi/8}{1-\pi/8}$$

$$= \frac{8+\pi}{8-\pi}$$

2. (15 pts) Find

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^{(\sqrt[5]{x})}$$

$$y = (\ln x)^{(\sqrt[5]{x})}$$

$$\ln y = \sqrt[5]{x}\,\ln\ln x$$

$$\frac{y'}{y} = \frac{1}{5}x^{-4/5}\ln\ln x + \sqrt[5]{x}\,\frac{1}{x\ln x}$$

$$y' = \left(\frac{1}{5}x^{-4/5}\ln\ln x + \sqrt[5]{x}\,\frac{1}{x\ln x}\right)\ln x^{\sqrt[5]{x}}$$

$$= \left(\frac{\ln(\ln(x))}{5\sqrt[5]{x^4}} + \frac{\sqrt[5]{x}}{x\ln x}\right)\ln x^{\sqrt[5]{x}}$$

3. (15 pts) Use local linear approximation to approximate the value of $\sqrt[3]{30}$. Perform all arithmetic by hand, and use a graph to illustrate whether your approximation is an overestimate or an underestimate.

$f(x) = \sqrt[3]{x}$ $\quad f'(x) = \frac{1}{3}x^{-2/3}$

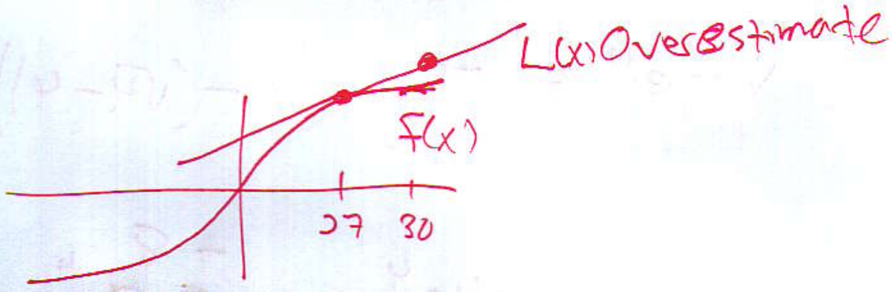
$a = 27$ $\quad f(a) = 3$ $\quad f'(a) = \frac{1}{27}$

$f(30) \approx f'(a)(x-a) + f(a)$

$= 3/27 + 3$

$= 3\,1/9$

$= 3.1111\ldots$



4. (15 pts) Find the equation for the tangent line to the curve with parametric equations

$$x = \sqrt{t} - t$$
$$y = e^t$$

when $t = 4$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t}{\frac{1}{2\sqrt{t}} - 1} = \frac{e^4}{\frac{1}{2\sqrt{4}} - 1}$$

$$= \frac{e^4}{\frac{1}{4} - 1} = \frac{e^4}{-3/4} = -\frac{4e^4}{3}$$

$$y - e^4 = \frac{-4e^4}{3}\left(x - (\sqrt{4} - 4)\right)$$

$$y = \frac{-4e^4}{3}x \; \frac{-8e^4}{3} + e^4$$

$$y = \frac{-4e^4}{3}x - \frac{5e^4}{3}$$

$$y \cong -72.798x - 90.997$$

5. (15 pts) Find y' for

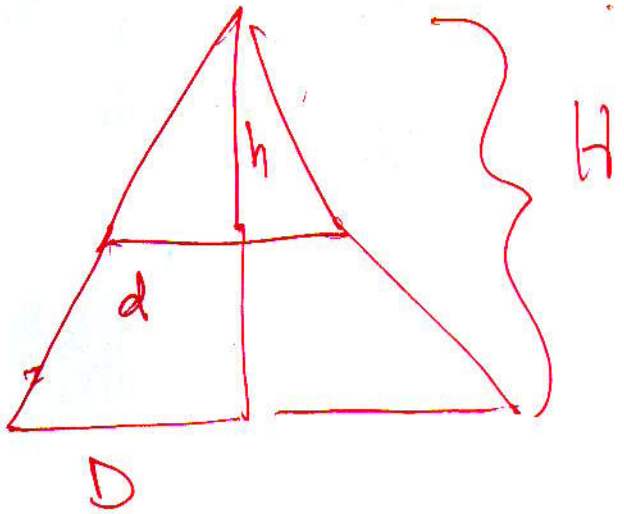
$$x^2 \sin\left(y^2\right) = 5xy$$

$$2x\sin(y^2) + x^2\cos(y^2)\,2y\,y' = 5y + 5xy'$$

$$y' = \frac{5y - 2x\sin(y^2)}{2x^2 y\cos(y^2) - 5x}$$

$$= \frac{2x\sin(y^2) - 5y}{5x - 2x^2 y\cos(y^2)}$$

6. (20 pts) Suppose a circular platform elevator is located directly below a spotlight and casts a shadow on the floor below. The height of the spotlight is 21 meters, the platform is 5 meters across, and its shadow is 15 meters across. If the shadow's diameter is currently shrinking at a rate of 1 m/s, find the current height of the elevator, determine whether the elevator is moving up or down, and find its speed. (Hint: draw a cross-section of the elevator with its shadow and use similar triangles.)



$H = 21$

$D = 15$

$d = 5$

d, H const

$$\frac{h}{H} = \frac{d}{D}$$

$$h = \frac{dH}{D} = \frac{5(21)}{15} = 7$$

So height is $21 - 7 = 14$ m

$$\frac{h'}{H} = \frac{-dD'}{D^2}$$

$$h' = \frac{-dHD'}{D^2} = \frac{-5(21)(-1)}{15^2} = 0.467 \text{ m/s}$$

h is getting larger, so elevator is moving down

7. (20 pts) The temperature T (in °F) of a 12" skillet over a gas flame is given as a function of the distance d (in inches) from the center of the skillet by

$$T(d) = -0.2d^3 - 1.5d^2 + 14.4d + 200$$

Find the maximum and minimum temperature of the skillet and where these temperatures occur.

$$T'(d) = -0.6d^2 - 3d + 14.4$$

$$d = \frac{3 \pm \sqrt{(-3)^2 - 4(-0.6)(14.4)}}{2(-0.6)}$$

$$= \frac{3 \pm \sqrt{9 + 34.56}}{-1.2}$$

$$= \frac{3 \pm 6.6}{-1.2} = \cancel{-8}, 3$$

center $T(0) = 200$

½-way out $T(3) = 224.3°F$ @max

rim $T(6) = 189.2°F$ @min