

Name: Grading Guide

Each problem is worth 15 or 20 points. Work at least four of the first five problems and both of the last two problems; you may work all seven problems for extra credit. Numerical estimates are unacceptable; for full credit you must show all your work and use the indicated methods.

1. (15 pts) Find the slope the tangent line to $r = \theta^2$ when $\theta = \frac{\pi}{4}$.

- 3 Doesn't sub $\theta = \pi/4$
- 3 Uses degrees
- 5 Gets basic setup right but doesn't take $x = r \cos \theta$, $y = r \sin \theta$
- 2 Arithmetic / transcription error
- 2 Reverses num & denom
- 12 Takes derivative to be $r' = 2\theta = \pi/2$
- 12 Gets $x = r \cos \theta$
 $y = r \sin \theta$ but not much else
- 10 Does something that looks vaguely like correct setup

2. (15 pts) Find

$$\frac{d}{dx}(\ln x)^{(\sqrt[5]{x})}$$

~~10000~~

- 12 Uses something like power rule - chain rule but no logarithmic diff, prod. rule, etc.
- 4 Doesn't use chain rule in computing $\frac{d}{dx} \ln x$
- 1 writes $\frac{d}{dx} =$
- 14 Uses something that doesn't look like an application of chain, power, product, or anything else
- 4 Does log diff but not mult. y
- 3 Messes up chain rule in $\frac{d}{dx} \ln x$
- 4 Does log diff but messes up details on log props

3. (15 pts) Use local linear approximation to approximate the value of $\sqrt[3]{30}$. Perform all arithmetic by hand, and use a graph to illustrate whether your approximation is an overestimate or an underestimate.

- 12 Does something like a difference quotient
- 4 Doesn't substitute $x = 30$
- 1 mislabels graph
- 3 Uses $a = 8$
- 3 ~~Graph~~ Graph doesn't look like tangent line, but says overest.
- 4 Gets $L(x)$ slightly wrong
- 2 Says underest
- 5 Uses a ~~not~~ not a perfect cube
- 6 Uses $a = 30$
- 6 Gets a formula for $L(x)$ that looks only vaguely reminiscent of the correct one

4. (15 pts) Find the equation for the tangent line to the curve with parametric equations

$$\begin{aligned}x &= \sqrt{t} - t \\ y &= e^t\end{aligned}$$

when $t = 4$.

-3 Doesn't sub $t = 4$

-7 Does $\frac{d}{dt} \frac{y}{x}$

-3 Gets $\frac{d}{dt} \{e^t, \sqrt{t} - t\}$ wrong

~~scribble~~

-2 Sign error

-4 Severe algebra

-4 Doesn't give final eqn

-1 ea Gets x or y coord wrong

-2 arithmetic

-2 Reverses num & denom

5. (15 pts) Find y' for

$$x^2 \sin(y^2) = 5xy$$

- 1 transcription error
- 3 minor alg
- 3 ca Doesn't use chain rule where appropriate
- 3 ca doesn't use product rule where appropriate
- 2 doesn't solve for y'
- 1 Sign error
- 4 major alg
- 13 doesn't use implicit diff

6. (20 pts) Suppose a circular platform elevator is located directly below a spotlight and casts a shadow on the floor below. The height of the spotlight is 21 meters, the platform is 5 meters across, and its shadow is 15 meters across. If the shadow's diameter is currently shrinking at a rate of 1 m/s, find the current height of the elevator, determine whether the elevator is moving up or down, and find its speed. (Hint: draw a cross-section of the elevator with its shadow and use similar triangles.)

- 8 Does implicit diff. on some halfway reasonable eqn but doesn't use similar triangles
- 4 Sign error on $\frac{dh}{dt}$
- 3 Doesn't differentiate const. correctly
- 3 Doesn't find current height
- 8 Doesn't use sim. tri. or anything remotely reasonable
- 9 No implicit diff
- 2 Doesn't give 21-h for ht

7. (20 pts) The temperature T (in $^{\circ}\text{F}$) of a 12" skillet over a gas flame is given as a function of the distance d (in inches) from the center of the skillet by

$$T(d) = -0.2d^3 - 1.5d^2 + 14.4d + 200$$

Find the maximum and minimum temperature of the skillet and where these temperatures occur.

- 5 Doesn't show work on $T'(d) = 0 \Rightarrow d = -8, 3$
- 1 minor alg.
- 7 Doesn't test endpoints (tests one - 4)
- 7 Doesn't find both max, min
- 5 Doesn't throw out $d = -8$
- 3 Thinks we're talking about a 2' skillet (her), says dom $T = [0, 12]$
- 2 Calculates d wrong from correct derivative
- 2 No units