Exam 3

Name: _____

Each problem is worth 15 or 20 points. Work at least four of the first five problems and both of the last two problems; you may work all seven problems for extra credit. Numerical estimates are unacceptable; for full credit you must show all your work and use the indicated methods.

1. (15 pts) Find the slope the tangent line to $r = \theta^2$ when $\theta = \frac{\pi}{4}$.

2. (15 pts) Find

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^{(\sqrt[5]{x})}$$

3. (15 pts) Use local linear approximation to approximate the value of $\sqrt[3]{30}$. Perform all arithmetic by hand, and use a graph to illustrate whether your approximation is an overestimate or an underestimate.

4. (15 pts) Find the equation for the tangent line to the curve with parametric equations

$$x = \sqrt{t} - t$$
$$y = e^t$$

when $t = 4$.

5. (15 pts) Find y' for

$$x^2 \sin\left(y^2\right) = 5xy$$

6. (20 pts) Suppose a circular platform elevator is located directly below a spotlight and casts a shadow on the floor below. The height of the spotlight is 21 meters, the platform is 5 meters across, and its shadow is 15 meters across. If the shadow's diameter is currently shrinking at a rate of 1 m/s, find the current height of the elevator, determine whether the elevator is moving up or down, and find its speed. (Hint: draw a cross-section of the elevator with its shadow and use similar triangles.)

7. (20 pts) The temperature T (in °F) of a 12" skillet over a gas flame is given as a function of the distance d (in inches) from the center of the skillet by

$$T(d) = -0.2d^3 - 1.5d^2 + 14.4d + 200$$

Find the maximum and minimum temperature of the skillet and where these temperatures occur.