

(Multiply total by 10/9)

Name: Grading Guide

Each problem is worth 15 points. Numerical estimates are unacceptable; for full credit you must show all your work and use the indicated methods.

1. Suppose the position of a particle after t seconds is given by $f(t) = t^3 - 3t^2 + 2t + 1$. Determine when the particle is speeding up and when it is slowing down.

- 4 Claims f' always > 0
- 6 Does correct numerical estimate
- 7 Checks sign of f'' but doesn't compare with f'
- 4 Compares signs but doesn't make correct table
- 4 Gets quadratic equation incorrect or produces a 'factorization' of $f'(x)$ over $\mathbb{Q}[x]$
- 8 Does extremely coarse numerical estimate, e.g. only checks integer x -values.
- 13 Nothing sensible except derivatives
- 3 Invalid form for answer

2. Find the equation for the highest horizontal tangent line to

$$f(x) = x^2 e^x$$

- 5 Doesn't set $f'(x) = 0$ to get $x = -2$
- 5 Doesn't give an eqn / y-value
- 3 Doesn't solve
For both roots of $f'(x) = 0$
- ~~0~~
- 01 Doesn't verify that $y = 4e^{-2}$ is highest.
- 1 Gives y-value, no eqn

3. Use the fact that $\frac{d}{dx} \sin x = \cos x$ and $\frac{d}{dx} \cos x = -\sin x$ to prove that $\frac{d}{dx} \tan x = \sec^2 x$.

- 1 Writes \sec^2
- 1 writes $\frac{1}{\cos^2 x} = \frac{1}{\sec^2 x}$
- 2 sign error

4. Use the definition of the derivative to find

$$\frac{d}{dx} \frac{1}{(4x+3)^2}$$

-10 Uses quotient rule (correctly)

-13 " " (incorrectly)

-1 sign error

-1 writes "lim" or " $\frac{d}{dx} =$ "

-7 doesn't show work on expanding binomials

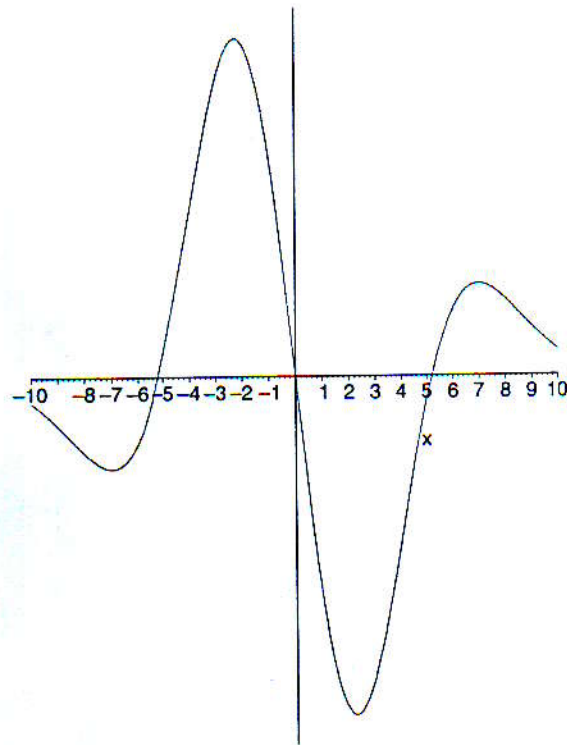
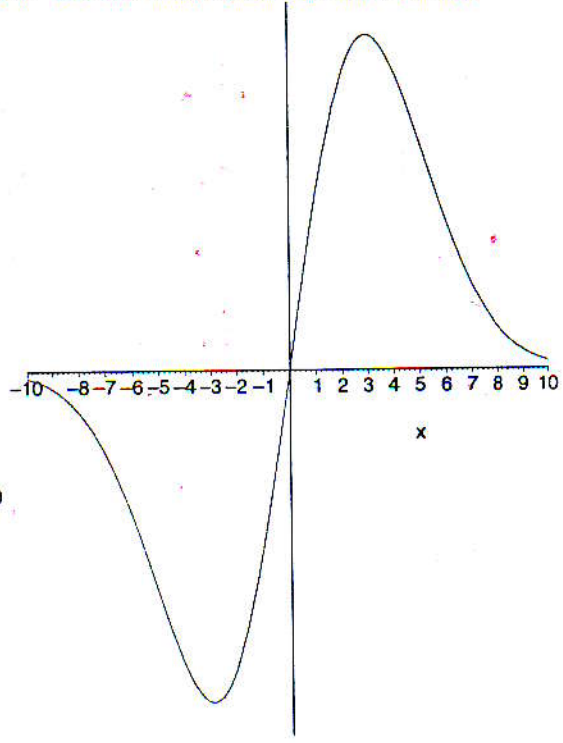
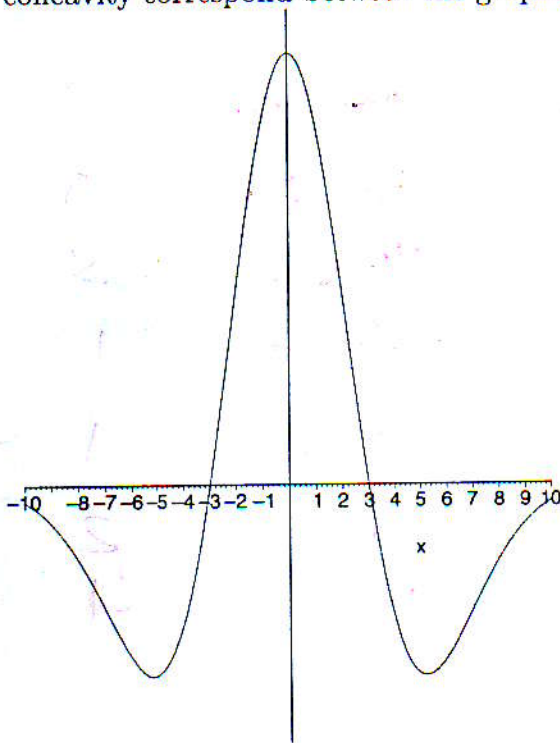
5. Suppose the temperature (in °C) of a point on an iron rod protruding from a kiln is given as a function of the distance (in cm) to the end by

$$T(d) = \begin{cases} 1500 & \text{if } 0 \leq x < 16 \\ \frac{3000\sqrt{x}}{\sqrt[3]{x^3}} & \text{if } x \geq 16 \end{cases}$$

Find $T'(81)$. Give units and interpret your answer.

- 2 Doesn't specify meaning of indep. var.
- 8 Doesn't take a derivative
- 7 No interpretation
- 5 Uses quotient rule & borks it up completely
- 6 Nonsense interpretation
- 2 Doesn't write $\frac{d}{dx}$ until applying power rule; does something like $T' = 3000x^{-1/4} = -\frac{1}{4}(3000)x^{-5/4}$
- 3 minor error using quotient rule
- 1 sign error in interp.

6. The functions f , f' , and f'' are pictured below. Label each graph with the appropriate function. Justify your answer by identifying how sign, slope and concavity correspond between the graphs of the function and its derivatives.



- 1 ea bit of nonsense
- 3 ea non-completed exp. req.
- 5 Writes something about antiderivatives
- 5 Gives intervals of sign, but doesn't relate them
- 3 ea wrong label