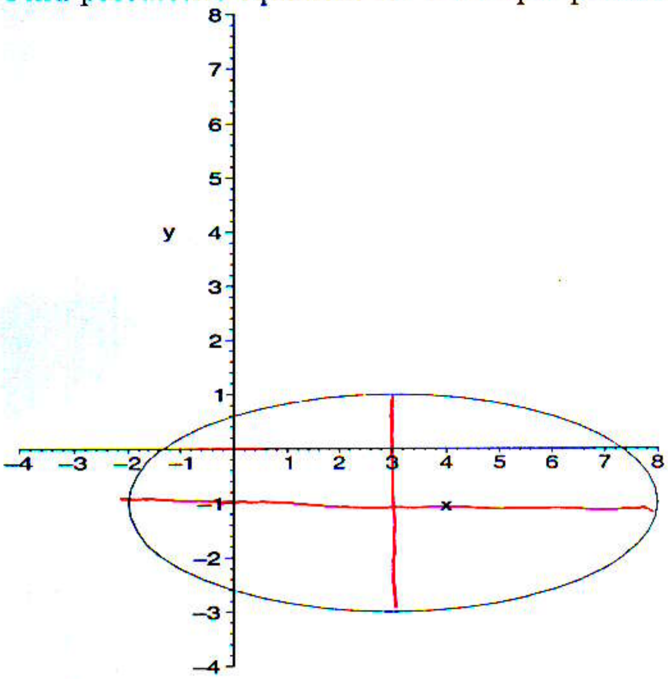
Name: ICGY

Each problem is worth 15 points. Show all your work.

1. Find parametric equations for the ellipse pictured below.



$x = 5\cos t + 3$

$y = 2\sin t - 1$

2. State the formal definition of the limit, and use it to show that

$$\lim_{x \to 0} x^4 = 0$$

Let $\varepsilon > 0$, set $\delta = \varepsilon^{1/4}$. Then

$0 < |x - 0| < \delta \Rightarrow |x^4 - 0| =$

$|x|^4 < \delta^4 = (\varepsilon^{1/4})^4 = \varepsilon.$

$\lim_{x \to a} f(x) = L \Leftrightarrow \forall\, \varepsilon > 0\ \exists\, \delta > 0 \ni$

$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

3. Let $f(x) = \frac{|x|}{x} + 4$ and $g(x) = -\frac{2|x|}{x} + 8$. For each of the limits listed below, find the limit or explain why it does not exist.

(i) $\lim_{x\to 0^+} f(x)$ 5

(ii) $\lim_{x\to 0^-} f(x)$ 3

(iii) $\lim_{x\to 0} f(x)$ DNE, (i) $\neq$ (ii)

(iv) $\lim_{x\to 0^+} g(x)$ 6

(v) $\lim_{x\to 0^-} g(x)$ 10

(vi) $\lim_{x\to 0} g(x)$ DNE (iv) $\neq$ (v)

(vii) $\lim_{x\to 0^+} (f+g)(x)$ 11

(viii) $\lim_{x\to 0^-} (f+g)(x)$ 13

(ix) $\lim_{x\to 0} (f+g)(x)$ DNE, (vii) $\neq$ (viii)

(x) $\lim_{x\to 0^+} (fg)(x)$ 30

(xi) $\lim_{x\to 0^-} (fg)(x)$ 30

(xii) $\lim_{x\to 0} (fg)(x)$ 30

4. Find

$$\lim_{x\to\infty} \frac{\cos x}{e^x}$$

$$\lim_{x\to\infty} \frac{1}{e^x} = 0 = \lim_{x\to\infty} \frac{-1}{e^x}$$

$$\frac{-1}{e^x} \le \frac{\cos x}{e^x} \le \frac{1}{e^x}$$

So $\lim_{x\to\infty} \frac{\cos x}{e^x} = 0$ by Squeeze Thm

5. Find

$$\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$$

$$\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \cdot \frac{\sqrt{9+h}+3}{\sqrt{9+h}+3} = \lim_{h\to 0} \frac{\cancel{9}+h-\cancel{9}}{h(\sqrt{9+h}+3)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)} = \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} = \frac{1}{6}$$

6. Approximate π to one place after the decimal using bisection on the function $f(x) = \sin x$.

a	$\frac{a+b}{2}$	b	$\sin(a)$	$\sin\left(\frac{a+b}{2}\right)$	$\sin(b)$
3	3.5	4	0.14	−0.35	−0.75
3	3.125	3.5	0.14	0.0166	−0.35
3.125	3.3125	3.5	0.0166	−0.17	−0.35
3.125	3.219	3.3125	0.0166	−0.077	−0.17
3.125	3.171875	3.219	0.0166	−0.0302	−0.077
3.125	3.148	3.171875	0.0166	−0.006	−0.0302
3.125	3.137	3.148	0.0166	0.0049	−0.006

$\pi \approx 3.1$

7. Show that $f(x) = \begin{cases} x^2 & x \geq 2 \\ 5x-6 & x < 2 \end{cases}$ is continuous at $x = 2$.

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 = 2^2 = 4$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (5x-6) = 5(2)-6 = 4$

~~f(2)~~

So $\lim_{x \to 2} f(x) = 4$

and $f(2) = 2^2 = 4$

Hence $f(x)$ is cont at $x = 2$