

Name: _____

The last problem is worth 75 points; each of the other problems is worth 25 points. Complete the last problem and at least three of the other four; you may complete an additional problem for extra credit.

1. Find all maxima, minima, and inflection points of

$$f(x) = 3x^4 + 4x^3$$

2. Use Newton's method to find the zero of $f(x)$ closest to the origin, where

$$f(x) = e^x - \sin x$$

Write out the complete statement of Newton's method for the first two approximations after your initial guess, and write out the value of all successive approximations until the sequence converges to within the maximum accuracy of your calculator. (Be careful with your initial guess here. You should probably use a graph to make sure you really get the closest zero).

3. Find

$$\lim_{x \rightarrow \infty} \left[\cot\left(\frac{\pi}{x}\right) - \csc\left(\frac{\pi}{x}\right) \right]$$

4. Find

$$\lim_{x \rightarrow 1^+} [x^{-\ln(x-1)}]$$

5. A security guard at a nuclear power plant is trapped between the double layer of electric fencing surrounding the perimeter of the facility during a meltdown. The entire complex runs 3 kilometers from east to west and 1 kilometer from north to south, with reactors located 500 meters from the east and west ends. Before dying, his coworkers at the security stations located 500 meters due south of the reactors reported radiation levels of 600 rems near the eastern reactor and 300 rems near the western reactor. Due to his location at the central south gate, the guard has managed to survive. If the guard can move freely from east to west inside the fence, determine where he should stand relative to his current position to minimize his radiation exposure until help arrives. You may use your calculator to find the zeros of any equations as needed. (Use the fact that radiation intensity is inversely proportional to the square of the distance from its source).