Exam 1

Name: _____

Each problem is worth 25 points. Complete at least six problems; you may complete the seventh for extra credit. Show all your work.

1. Show that

$$\lim_{x\to\infty}\left(\frac{1}{x}cosx\right)=0$$

2. Find

$$\lim_{x\to\frac{\pi}{2}}\frac{1}{1+\tan^2 x}$$

3. Use limits to find all asymptotes of

$$\frac{x^2 - 4}{x^2 + 8x + 16}$$

4. State the definition of continuity, and use the definition to show that $f(x) = \sqrt{|x|}$ is continuous at $x = 0$.

5. Find the value of the following limits for the functions $f(x)$ and $g(x)$ pictured below:

(i) $\lim_{x\to 0^+} f(x)$

(ii) $\lim_{x\to 0^-} f(x)$

(iii) $\lim_{x\to 0} f(x)$

(iv) $\lim_{x\to 0^+} g(x)$

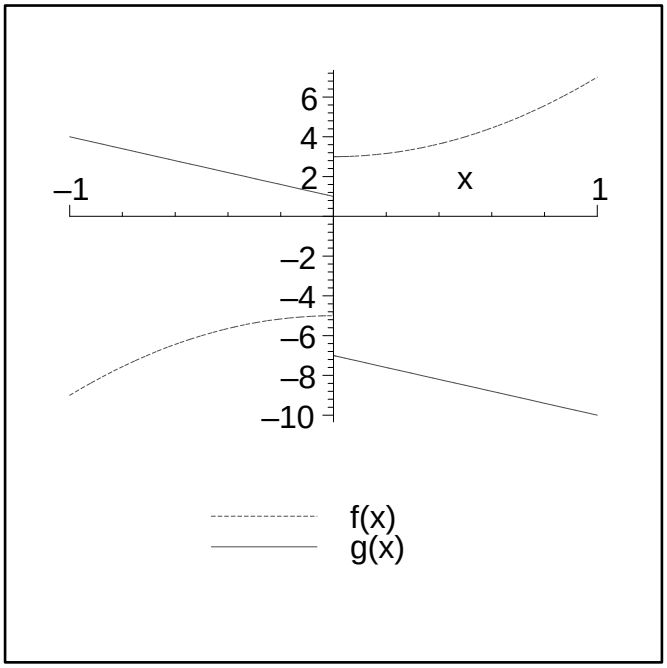
(v) $\lim_{x\to 0^-} g(x)$

(vi) $\lim_{x\to 0} g(x)$

(vii) $\lim_{x\to 0^+} ((f+g)(x))$

(viii) $\lim_{x\to 0^-} ((f+g)(x))$

(ix) $\lim_{x\to 0} ((f+g)(x))$



6. Use the definition of the limit to show that

$$\lim_{x \to \infty} x^2 = \infty$$

(Recall that $\lim_{x \to \infty} f(x) = \infty$ means that for all $M > 0$, there exists $N > 0$ such that $x > N \Rightarrow f(x) > M$.)

7. Use trigonometric identities to show that $x = 2\cos t$, $y = 5\sin t$ are the parametric equations for an ellipse centered at the origin.