Name: KEY

Each problem is worth 25 points. Complete at least six problems; you may complete the seventh for extra credit. Show all your work.

1. Show that

$$\lim_{x\to 1}\left(\underbrace{\frac{x^3-3x^2+3x-1}{x-1}\sin\left(\frac{\pi}{x-1}\right)}_{f(x)}\right)=0$$

$$\lim_{x\to 1}\frac{x^3-3x^2+3x-1}{x-1}=\lim_{x\to 1}\frac{(x-1)^3}{x-1}=\lim_{x\to 1}(x-1)^2=0$$

So $\lim_{x\to 1} -\left(\frac{x^3-3x^2+3x-1}{x-1}\right)=0$

$$-1\le \sin\frac{\pi}{x-1}\le 1$$

$$-\left(\frac{x^3-3x^2+3x-1}{x-1}\right)\le \frac{x^3-3x^2+3x-1}{x-1}\sin\frac{\pi}{x-1}\le\frac{x^3-3x^2+3x-1}{x-1}$$

$\Rightarrow \lim_{x\to 1} f(x)=0$ by the squeeze Th^m.

2. Find

$$\lim_{x\to 0^+}\frac{1}{\ln x}$$

$$\lim_{x\to 0^+}\ln x=-\infty$$

$\frac{1}{y}$ is conts for $y<0$, so

$$\lim_{x\to 0^+}\frac{1}{\ln x}=\lim_{y\to -\infty}\frac{1}{y}=0$$

3. Use limits to show that

$$\frac{x^3 - x^2 + x - 1}{x^2 + 8x + 16} = f$$

has no horizontal asymptotes.

$$\lim_{x \to \infty} \frac{x^3 - x^2 + x - 1}{x^2 + 8x + 16} = \lim_{x \to \infty} \frac{x - 1 + 1/x - 1/x^2}{1 + 8/x + 16/x^2} = \infty$$

Likewise

$$\lim_{x \to -\infty} \frac{x^3 - x^2 + x - 1}{x^2 + 8x + 16} = -\infty$$

So $f(x)$ has no finite limits at ∞ and thus no hori. asym.

4. Determine where $f(x) = \frac{\sqrt{\ln(x+4)}}{2^x - 2}$ is continuous. (on its domain)

$x + 4 > 0$

$\ln(x+4) \geq 0$

$x + 4 \geq 1$

$x \geq -3$

$2^x \neq 2$

$x \neq \log_2 2 = 1$

$[-3, 1) \cup (1, \infty)$

5. Find the value of the following limits for the functions $f(x)$ and $g(x)$ pictured below:

(i) $\lim_{x\to 0^+} f(x)$ = 3

(ii) $\lim_{x\to 0^-} f(x)$ = -5

(iii) $\lim_{x\to 0} f(x)$ DNE

(iv) $\lim_{x\to 0^+} g(x)$ = -7

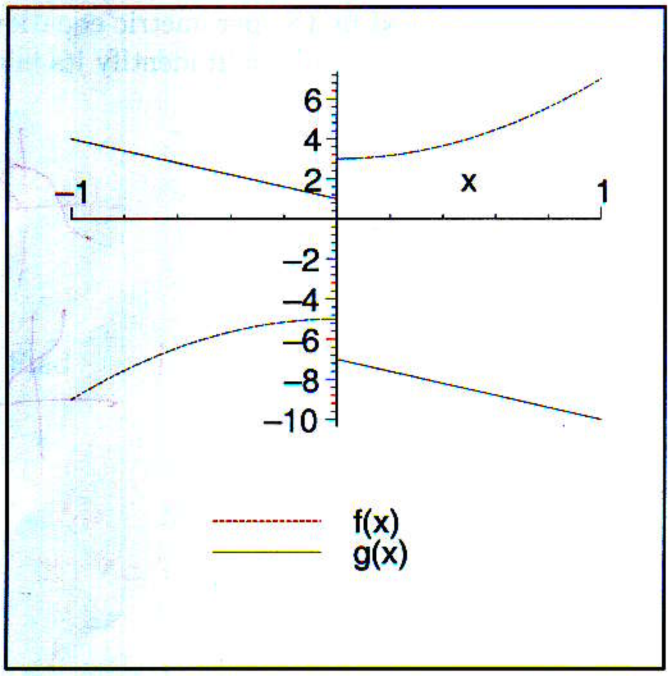
(v) $\lim_{x\to 0^-} g(x)$ = 1

(vi) $\lim_{x\to 0} g(x)$ DNE

(vii) $\lim_{x\to 0^+} (f(x) + g(x))$ = -4

(viii) $\lim_{x\to 0^-} (f(x) + g(x))$ = -4

(ix) $\lim_{x\to 0} (f(x) + g(x))$ = -4



6. State the definition of the limit, and use the definition to prove that

$$\lim_{x\to -3}(4-3x)=13$$

$\lim_{x\to a} f(x)=L \iff \forall\, \varepsilon>0\ \exists\, \delta>0 \ni$

$0<|x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon.$

Let $\varepsilon>0$, set $\delta=\varepsilon/3$. Then

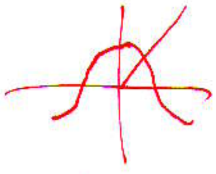
$0<|x--3|<\delta \Rightarrow |x+3|<\delta \Rightarrow$

$|4-3x-13| = |-3x-9| = 3|x+3| < 3\delta =$

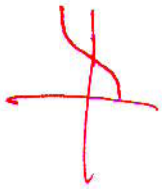
$3\varepsilon/3 = \varepsilon.$

7. Find a Cartesian equation for the curve determined by the parametric equations $x = 3\cos t - 1, y = 3\sin t - 2, 0 \le t \le 2\pi$. Sketch the curve and identify its initial point and terminal point.

$\frac{x+1}{3} = \cos t \qquad \frac{y+2}{3} = \sin t$

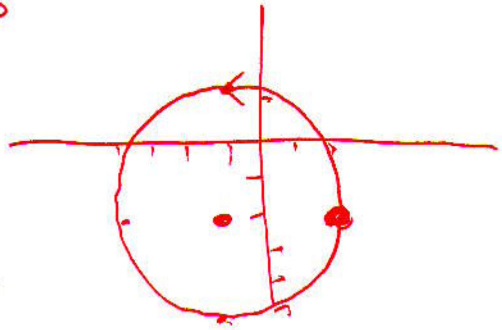


$\left(\frac{x+1}{3}\right)^2 + \left(\frac{y+2}{3}\right)^2 = \cos^2 t + \sin^2 t = 1$



$(x+1)^2 + (y+2)^2 = 3^2$

$(x-(-1))^2 + (y-(-2))^2 = 3^2$



initial pt $= (2,-2)$

$=$ terminal pt.