Exam 3

Name: ______________________

Each problem is worth 20 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested. Work both of the first two problems, at least two of problems 3-6, and at least one of problems 7-9. You may work one additional problem for 10 points of extra credit; make sure you indicate which problem is to be graded as a bonus. If you work more than five problems, I will only grade 1-5 and 7.

1. Use Newton's Method to approximate the zero of $f(x) = \ln(x) - 1$ closest to $x = 3$ to the maximum accuracy supported by your calculator. What is the exact value of the number you are approximating?

2. Graph by hand:

$$f(x) = \frac{(x-1)^2}{x+2}$$

Show all intercepts, asymptotes, extrema, inflection points, intervals of increase/decrease, and intervals of concavity.

3. Find

$$\lim_{x \to 2^+} \frac{x^2 - 4}{x^3 - 8}$$

4. Find

$$\lim_{x\to\infty} \sqrt{x^2+4x-5} - \sqrt{x^2+5x+3}$$

5. Find

$$\lim_{x\to\infty} \ln(x) \ln\left(\frac{1}{x}\right)$$

6. Find

$$\lim_{x\to\infty} x^{\left(\ln\left(\frac{1}{x}\right)+1\right)}$$

7. Find the coordinates of the points on the graph of $f(x) = \cos x$ closest to the origin. (You may find critical numbers graphically, but the rest of the problem must be done by hand.)

8. Find the largest rectangle that can be contained in an ellipse with long radius 4 and short radius 3, assuming the sides of the rectangle are parallel to the axes of the ellipse.

9. Suppose a packing crate is to be constructed in the shape of a right hexagonal prism with regular bases (the top and bottom are identical hexagonal pieces of material in which all the interior angles and edge lengths are equal, the sides are rectangles, and the angles between any side and the top or bottom is a right angle). If the crate must have a capacity of 5ft^3, find the dimensions that minimize the amount of material used in its construction.