Exam 3

Name: ______________________

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested. Work all of the first six problems and at least one related rates problem (problems 7-9). You may work one additional related rates problem for extra credit (if you complete all three, I will only grade the first two).

1. Differentiate:

   (a) $\sin(\ln(x))$

   (b) $\ln(\sin(x))$

   (c) $\ln x$

   (d) $(\sin x)^{(\ln x)}$

2. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if

$$\sqrt[4]{x-y} = \ln(xy)$$

3. Determine where the tangent line to

$$r = \cos\theta$$

is horizontal and where it is vertical.

4. State and prove the differentiation rule for $y = \tan^{-1} x$.

5. Differentiate:

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{e^{\sqrt{x}}\ln\left(x\right)\left(x^2-5x+3\right)^{10}}{\sin x\cos x}$$

6. Find all global extrema of $f(x) = 3x^4 + 4x^3 - 72x^2 + 300$ on $[-3, 3]$.

7. A hemispherical bowl 10cm in diameter with a hole in the bottom sits on the surface of a pond. Water leaks in so that the depth of the water in the bowl is increasing by 1cm/min. Determine how quickly the surface area of the water inside the bowl is increasing when the depth is 3cm. (Consider only the upper surface of the water.)

8. A 5ft tall woman stands between a red traffic light on top of a 10ft pole and a green traffic light on top of a 15ft pole. The traffic lights are 40ft apart. (She casts a red shadow in the direction of the red light and a green shadow in the direction of the green light, but is otherwise surrounded by yellow light.) If the total length of her shadows is increasing by 2ft/s, determine which light she is moving towards, and how quickly she is moving.

9. Suppose the volume of a cylinder is decreasing by $20\pi \text{cm}^3/\text{s}$ while its surface area (with end caps included) is increasing by $8\pi \text{cm}^2/\text{s}$. Determine how quickly its radius and height are changing when the radius is 20cm and the height is 8cm.