Exam 2

Name: ______________________

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested. Work at least seven problems; you may work an eighth for extra credit (if you complete more than eight, I will only grade the first eight).

1. Find the derivative:

   (a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[e^x + x^e + ex^e + xe^x\right]$

   (b) $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt[5]{x^3}$

   (c) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{\sin x}{x^4}$

2. Find

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{x^3-2x^2-20x+15}{x+4}$$

(a) Using the definition of the derivative.

(b) Using the quotient rule.

Check that your answers agree.

3. Differentiate:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[e^x \tan(x)\sqrt{x}\right]$$

4. State the product rule, and prove the product rule using the definition of the derivative.

5. State the differentiation rule for $f(x) = \cos x$, and prove this differentiation rule using the definition of the derivative. You may assume the following:

   (i) $\lim_{h \to 0} \frac{\sin h}{h} = 1$

   (ii) $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$

   (iii) $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

6. Use the differentiation rules for $\sin x, \cos x$, and the quotient rule to prove the differentiation rule for $\cot x$.

7. Find the equation for the tangent line to the graph of $f(x) = \sec x$ at the point $(0, 1)$.

8. Let $f(x) = x^4 + 4x^3$.

   (a) Find the equation for the tangent line to the graph of $f(x)$ at the point $(2, 48)$

   (b) There is a tangent line to the graph of $f(x)$ at a point $x \neq 2$ that intersects the graph a second time at $(2, 48)$. Find the equation of this tangent line.

9. Suppose the position of a particle on the $y-$axis at time t is given by

$$D(t) = 2t^3 - 45t^2 + 300t$$

Determine when the particle is speeding up and when it is slowing down.

10. Suppose the area (in m^2) inside a mushroom ring t years after germination of a spore is given by $A(t) = \pi \left(\frac{100t}{10+t}\right)^2$. Find $A(5)$ and $A'(5)$. Give units and interpret your answer.