Name: KEY

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested. Several of the problems have a bonus component; you may attempt up to 10 points of bonus problems (if you complete more, I will only grade the first 10 points worth).

1. Graph each of the following sets of parametric equations by eliminating the parameter to get a Cartesian equation. Explain the difference between how the two parametric curves are traced out.
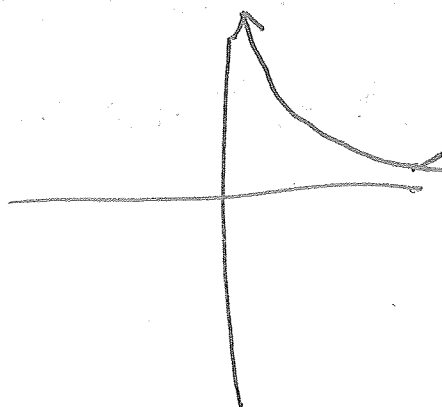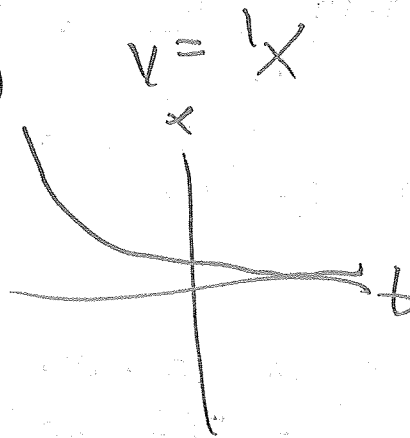
(a) $\begin{cases} x = \tan t \\ y = \cot t \end{cases}$ $(0 < t < \pi)$

(b) $\begin{cases} x = e^{-t} \\ y = e^t \end{cases}$

(a) $y = 1/x$
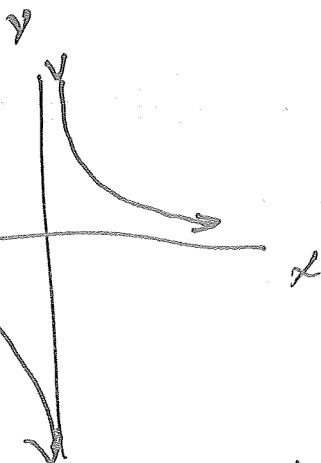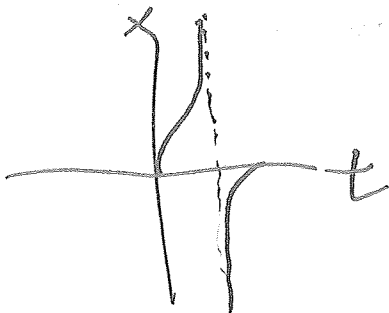
(b) $y = \dfrac{1}{x}$
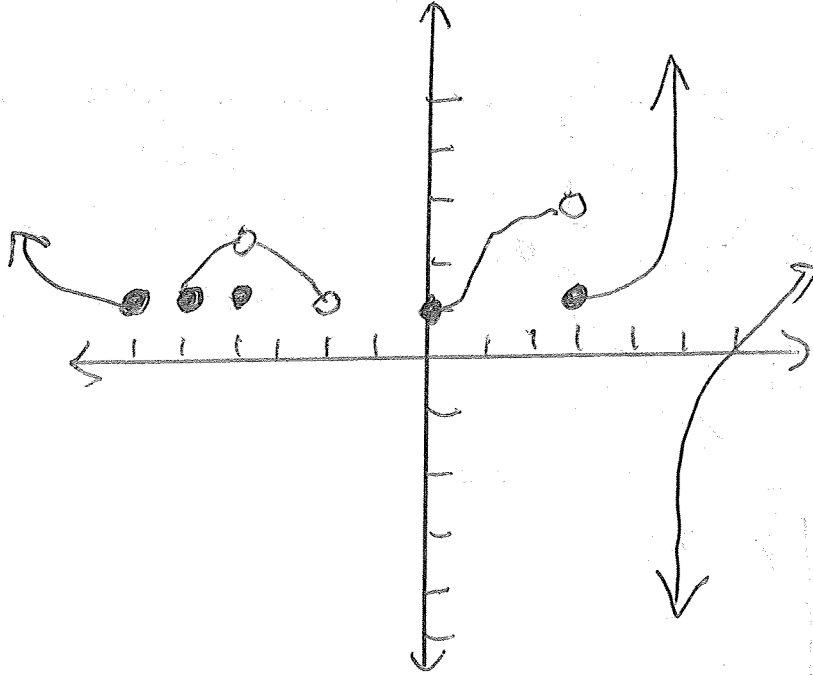

All possible x-values
L to R @ ➀ @ ➁, right half
traced out first

positive x only R to L

3. Use a graph to give an example of a pair of functions $f(x)$ and $g(x)$ such that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ do not exist but $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists. For 5 bonus points, give formulas for $f(x)$ and $g(x)$.

$$f(x) = g(x) = |x|/x$$

5. Determine the intervals on which the graph of $f(x)$ pictured below is continuous.

$$(-\infty, -6] \cup [-5, -4) \cup (-4, -2) \cup [0, 3) \cup$$

$$(3, 5) \cup (5, \infty)$$

7. State the formal definition of the limit, and use the definition to show that

$$\lim_{x \to 2} (2x + 3) = 7$$

For 5 bonus points, draw a graph and label what the variables $a, L, \delta,$ and ϵ represent for the above limit.

Let $\epsilon > 0$, set $\delta = \epsilon/2$. Then

$$0 < |x - a| < \delta \implies$$

$$-\epsilon/2 < x - 2 < \epsilon/2 \implies$$

$$-\epsilon/2 + 2 < x < \epsilon/2 + 2 \implies$$

$$-\epsilon + 4 < 2x < \epsilon + 4 \implies$$

$$-\epsilon + 7 < 2x + 3 < \epsilon + 7 \implies$$

$$-\epsilon + 7 < y < \epsilon + 7 \implies$$

$$-\epsilon < y - 7 < \epsilon \implies$$

$$|y - 7| < \epsilon$$

$$L + \epsilon$$
$$L = 7$$
$$L - \epsilon$$

$$a - \delta \quad a + \delta$$

$$a = 2$$