Exam 1

Name: _____

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested. Several of the problems have a bonus component; you may attempt up to 10 points of bonus problems (if you complete more, I will only grade the first 10 points worth).

1. Graph each of the following sets of parametric equations by eliminating the parameter to get a Cartesian equation. Explain the difference between how the two parametric curves are traced out.

   (a) $\begin{cases} x &= \tan t \\ y &= \cot t \end{cases}$ $(0 < t < \pi)$

   (b) $\begin{cases} x &= e^{-t} \\ y &= e^{t} \end{cases}$

2. Find at least two of the following limits (5 bonus points each for the other two):

   (a) $\lim_{x\to 10} \frac{x^2+x-110}{x-10}$

   (b) $\lim_{h\to 0} \frac{\sqrt{36+h}-6}{h}$

   (c) $\lim_{h\to 0} \frac{(t+h)^{-1}-t^{-1}}{h}$

   (d) $\lim_{x\to 7} \frac{x^4-12x^3+45x^2-74x+28}{x-7}$

3. Use a graph to give an example of a pair of functions $f(x)$ and $g(x)$ such that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ do not exist but $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists. For 5 bonus points, give formulas for $f(x)$ and $g(x)$.

4. Show that

$$\lim_{x \to 0} \left(x^2 \sin \frac{\pi}{x} \right) = 0$$

State the limit law(s) you are using.

5. Determine the intervals on which the graph of $f(x)$ pictured below is continuous.

6. Find

$$\lim_{x \to -\infty} \left[\frac{\sqrt{9x^6 - 5}}{\sqrt[3]{8x^9 + 2}} \right]$$

7. State the formal definition of the limit, and use the definition to show that

$$\lim_{x \to 2} (2x + 3) = 7$$

For 5 bonus points, draw a graph and label what the variables a, L, δ, and ϵ represent for the above limit.